The power of a control qubit in weak measurements

Raul Coto[1], Víctor Montenegro[1,3], Vitalie Eremeev[2], Douglas Mundarain[4], Miguel Orszag[1]
[1]*Instituto de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago, Chile*
[2]*Facultad de Ingeniería, Universidad Diego Portales, Av. Ejercito 441, Santiago, Chile*
[3]*Department of Physics and Astronomy, University College London, London WC1E 6BT, UK*
[4]*Departamento de Física, Universidad Católica del Norte, Casilla 1280, Antofagasta, Chile*
(Dated: September 12, 2016)

In the late 80s, a curious effect suggested by Aharanov, Albert and Vaidman [1] opened up new vistas regarding quantum measurements on weakly coupled systems. There, a combination of a "weak" finite interaction together with a "strong" post-selection measurement leads to an anomalous effect, namely the mean value of a spin-1/2 particle in the $z-$direction lies outside the conventional spectrum of ± 1. Despite being just a theoretical curiosity, the achieved amplification could be useful in the realm of sensoring modest quantities below the standard quantum limit, where they would not be able to be detected otherwise. Hence, the accurate quantum control of the weak value amplification becomes highly essential for quantum sensoring and detection.

In this paper, we investigate the quantum control of the weak value amplification of a qubit system coupled to a meter, via a second non-interacting qubit, initially quantum correlated with the first one. Our results show that for weak measurements, the control can be remotely realized via the post-selected state of the second qubit or the degree of squeezing of the meter. Additionally, in a step towards the study of the quantum control of the amplification, we can easily manipulate the degree of quantum correlations between the initial correlated qubits. We find that the degree of Entanglement has no effect on the quantum control of the amplification. However, we have found a clear connection between the amplification and quantum discord like measurements as well as classical correlations between the qubits. Moreover, we generalize the analysis to two control qubits and we can conclude that the single control qubit scheme is more efficient. Lastly, we suggest an original application of the amplification control protocol on the enhancement of the quantum measurement accuracy, e.g. measuring the relative phase of the post-selected control qubit in a more precise way, as opposed to the no-amplification case.

Over the past three decades, important advances have been made using characteristics of light beams or matter to control the evolution of atomic and molecular systems. For instance, the development of new and highly coherent laser sources allow to control molecules in the ground state [2]. However, in order to have control over a quantum system is not compulsory to involve external fields, recently were proposed many alternative methods to control [3–6] and even drive the system to a target state [7, 8]. Quantum control of physical systems has been a central issue in recent quantum technology in relation to measurement-based processes [9], like for example entangling mechanical motion to microwave radiation [10], so for two physical systems, measurement of one system can determine the state of the other. An interesting control mechanism was recently proposed in an optomechanical system, to control the quantum state of light (single photons) using mechanical variables to monitor a beam splitter [11], which goes beyond the usual goals of this type of systems, that uses light to control a mechanical resonator.

Quantum Measurement Theory is as old as Quantum Mechanics. The collapse of quantum states in the measurement process, one of the basic assumptions in quantum mechanics and put forward by von Neumann in 1932 [12], strongly modifies such a state. The question then arises: what would happen if the interaction responsible for the measurement becomes weaker and weaker? For weak measurements (WM), a theory was developed by Aharonov and collaborators [1], where the strong impact of the measurement is drastically reduced. It consists in a gradual accumulation of information during a finite interaction time between the meter and the system. As a matter of fact, the state is hardly changed and after such a measurement the system is left in a state that in general is not an eigenstate of the observable we are trying to measure, which seems to contradict the basic principles of Quantum Mechanics. However, this is not so, since the information obtained after one event is so modest, that many measurement processes are necessary to actually get information on the system.

In the seminal paper [1], Aharonov, Albert and Vaidman (AAV) showed that the combination of a weak measurement followed by a strong post-selection measurement may lead to some strange effect, usually referred to as an anomalous Weak Value Amplification (WVA), anomalous in the sense that the inferred mean value of the measured system variable lies outside its range of eigenvalues. The AAV results have been discussed in many papers [13–17] and also experiments have been realized and confirmed their predictions [18, 19]. More, recently, ultra sensitive measurements have been performed [20], as well as precision metrology [21] and an exciting experiment on the observation of the average trajectories of single photons in a two-slit interferometer [22].

In the framework of the AAV approach, the present work proposes to clarify and resolve three research tasks, which are very important for theory and experiments in the Quantum Information Science. The main task is devoted to the

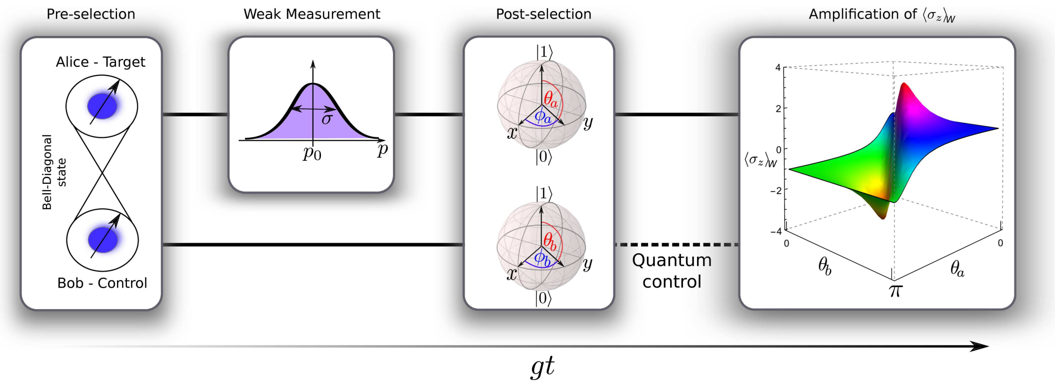

FIG. 1: Model of weak measurement amplification assisted by quantum correlated qubits.

effect of control of a quantum system using the *correlations* as resources in the processes of weak measurements. The second task clarifies which kind of correlations are indispensable when the WVA occurs. And the third task deals with the problem of enhancing the amplification effect by squeezing the meter state, making the interaction even weaker. In the following we present our results in detail.

Results

Model of Weak Value Amplification assisted by entangled qubits. Let us consider two qubits (a and b), initially prepared in a Bell Diagonal (BD) state, ρ^Q, such that one of them (a) interacts dispersively with a meter, ρ^M. The second qubit (b) does not interact at all and is only linked to the system via the quantum correlations existing between the two qubits, see Fig.(1). The Hamiltonian in the interaction picture is

$$H = \hbar g \sigma_3^a x, \tag{1}$$

where g is the coupling strength between the qubit a and the meter; σ_3 is the usual spin-1/2 Pauli operator in the $z-$direction and x denotes the continuous position of the meter. The initial state of the whole system, i.e., the two qubits together with the meter state is

$$\rho(0) = \rho^Q \otimes \rho^M = \frac{1}{4}(\mathbb{I} + \sum_{j=1}^{3} c_j \sigma_j^a \otimes \sigma_j^b) \otimes |\phi\rangle\langle\phi|, \tag{2}$$

where $\mathbb{I}$ is the identity operator in the two-qubit basis, σ_j are the Pauli operators and $|c_j| \leq 1$ are parameters satisfying the positivity of the density matrix. As known, BD states are defined by a set of three parameters $\{c_1, c_2, c_3\}$ depicted in a three dimensional tetrahedron, a geometrical representation of the subsets of entangled, separable and classical states [23–25].

Certainly, from the quantum measurement theory, the state of the meter must be expanded in the opposite conjugate variable appearing in Eq. 1, in our case the momentum subspace $|\phi\rangle = (2\pi\sigma^2)^{-1/4}\int_{-\infty}^{\infty} dp\,|p\rangle e^{-\frac{(p-p_0)^2}{4\sigma^2}}$, where σ and p_0 are the width and the center of the Gaussian profile, respectively. Subsequent the time evolution, we proceed to post-select the target state using a generic qubit state in the Bloch sphere as $|\psi_a\rangle = \cos(\theta_a/2)|1\rangle_a + \sin(\theta_a/2)e^{i\phi_a}|0\rangle_a$ (see Fig. 1). Notice that $|1\rangle$ and $|0\rangle$ are eigenstates of σ_3 with eigenvalues 1 and -1, respectively. To calculate the post-selected state of the system $\rho_{\psi_a} = \langle\psi_a|\rho(t)|\psi_a\rangle$, we make use of the usual translational operator in quantum mechanics, $e^{-igtx}|p\rangle = |p - gt\rangle$. Using the above equations and some algebra one gets

$$\begin{aligned}\rho_{\psi_a} &= \frac{1}{4\sigma\sqrt{2\pi}}\int dp\,dp'\,e^{-\frac{(p-p_0)^2}{4\sigma^2}-\frac{(p'-p_0)^2}{4\sigma^2}}\{\cos^2(\theta_a/2)\rho_{11}^Q|p-gt\rangle\langle p'-gt| + \sin^2(\theta_a/2)\rho_{00}^Q|p+gt\rangle\langle p'+gt| \\ &+ \cos(\theta_a/2)\sin(\theta_a/2)[\rho_{10}^Q e^{-i\phi_a}|p-gt\rangle\langle p'+gt| + h.c.]\},\end{aligned} \tag{3}$$

with $\rho_{11}^Q = {}_a\langle 1|\rho^Q|1\rangle_a = \mathbb{I}^b + c_3\sigma_3^b$, $\rho_{00}^Q = {}_a\langle 0|\rho^Q|0\rangle_a = \mathbb{I}^b - c_3\sigma_3^b$, and $\rho_{10}^Q = {}_a\langle 1|\rho^Q|0\rangle_a = c_1\sigma_1^b - ic_2\sigma_2^b$, and $\mathbb{I}^b$ is the identity operator in the b-qubit basis.

According to the Eq.(13) in the Sec. Methods, one can easily observe that by measuring a meter variable one can indirectly evaluate the weak value of the system variable of interest. Because of this, after the post-selection, we are interested in the expectation value of the momentum, which can be found by tracing over the meter degrees of

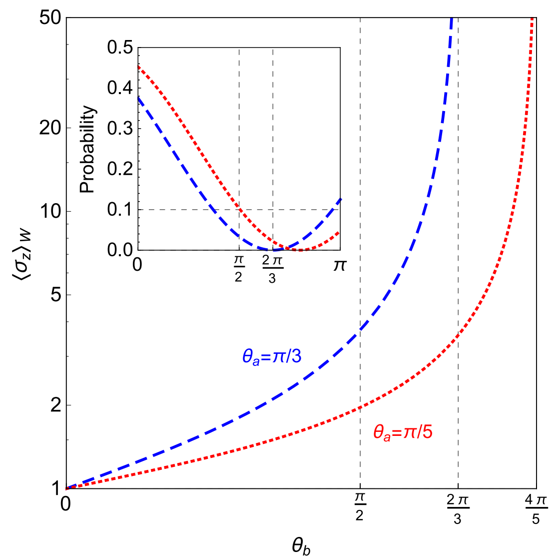

FIG. 2: The weak value amplification for a *Bell* state, i.e. Eq.(7), managed by the projections of the target qubit a, and control qubit b, with a given probability (Inset). Here $\delta \equiv \phi_a + \phi_b = \pi$ and $\sigma \to \infty$.

freedom. To investigate the effect of the control qubit in the amplification process, we shall leave the momentum expectation value expression as a function of the operators acting on the control qubit b. Furthermore, we would like to stress that, since the control qubit b does not interact with target qubit a nor with the meter, then the specific time at which one acts on b will not affect the quantum dynamics.

Next, in order to calculate

$$\langle p \rangle \equiv \frac{\langle Tr_M(\rho_{\psi_a} p)\rangle_b}{\langle Tr_M(\rho_{\psi_a})\rangle_b} \tag{4}$$

we derive, after some simple algebra, an expression for $Tr_M(\rho_{\psi_a} p)$, yielding the following

$$Tr_M(\rho_{\psi_a} p) = \frac{1}{4}[(\mathbb{I}^b + c_3\sigma_3^b)\cos^2(\theta_a/2)K_{11} + (\mathbb{I}^b - c_3\sigma_3^b)\sin^2(\theta_a/2)K_{00} + (c_1\sigma_1^b\cos\phi_a + c_2\sigma_2^b\sin\phi_a)\sin\theta_a K_{10}], \tag{5}$$

where the integrals K_{ij}, see Methods Eq.(17), are found to be $K_{11} = p_0 - gt$, $K_{00} = p_0 + gt$, $K_{10} = K_{01} = p_0 e^{-g^2t^2/2\sigma^2}$. The expression for $Tr_M(\rho_{\psi_a})$, denominator in Eq.(4), is calculated in a similar way. In fact, the expression is the same as above, by just replacing K_{ij} by J_{ij}, with $J_{11} = J_{00} = 1$ and $J_{10} = J_{01} = e^{-g^2t^2/2\sigma^2}$.

As mentioned above, one tries to understand the role of the control qubit b in the amplification process. To study this, let us consider two different approaches. (i) Firstly one traces over the control qubit b; (ii) Secondly one proceeds to perform a projection on the qubit b.

In the first case, considering Eq.(4), one gets $\langle p\rangle = p_0 - gt\cos\theta_a$. It is easy to see that this WM value does not lead to any amplification (independent of the initial condition) and the expectation value of the momentum is bounded by $p_0 \pm gt$. Furthermore, as known [26], coherence plays a significant role in the weak amplification process, thus when tracing over the qubit b, one eliminates the coherence in qubit a and therefore the amplification effect is gone.

In the second approach, one projects the control qubit b to a similar state as for the qubit a, i.e. $|\psi_b\rangle = \cos(\theta_b/2)|1\rangle_b + \sin(\theta_b/2)e^{i\phi_b}|0\rangle_b$, so calculating as in [1] the weak value for the spin operator, $\langle\sigma_z\rangle_W \equiv (p_0 - \langle p\rangle)/gt$ by using the Eq.(16) in the Sec. Methods. The expectation value corresponds to

$$\langle\sigma_z\rangle_W = \frac{c_3\cos\theta_b + \cos\theta_a}{1 + c_3\cos\theta_a\cos\theta_b + e^{-\frac{g^2t^2}{2\sigma^2}}\sin\theta_a\sin\theta_b(c_1\cos\phi_a\cos\phi_b + c_2\sin\phi_a\sin\phi_b)}. \tag{6}$$

This is the principal analytical result of our work for the model of one control qubit in WM. In the following we analyze some particular cases such as Bell and Werner states, and thereafter the general BD states.

Weak Value Amplification vs qubit correlations. This section is devoted to the study of the amplification effect via WM and the quantum correlations shared by the qubits. The amplification effect in the AAV model appears when the denominator in the weak value tends to zero, i.e. the pre- and post-selected states are almost orthogonal. Hence, as a simple and illustrative example let us consider the case of the two qubits initially prepared in a Bell state, e.g., $|\Phi^+\rangle = (|0_a 0_b\rangle + |1_a 1_b\rangle)/\sqrt{2}$ ($c_1 = c_3 = 1, c_2 = -1$) so Eq.(6) is then reduced to

$$\langle \sigma_z \rangle_W = \frac{\cos\theta_b + \cos\theta_a}{1 + \cos\theta_a \cos\theta_b + e^{-\frac{g^2 t^2}{2\sigma^2}} \sin\theta_a \sin\theta_b \cos\delta} \tag{7}$$

with $\delta = (\phi_a + \phi_b)$. Now, as in [1] if the meter state has a large Gaussian spread distribution on the momentum space, i.e. $\sigma \to \infty$, one can easily check that there are several different combinations of the projection angles that allow us to make the denominator as small as required. For example, we consider the set of angles $\{\delta = \pi, \theta_a + \theta_b = \pi\}$ which leads to a large amplification with the constraint $\theta_b \neq \{0, \pi\}$. This constraint comes from the simple fact that for these θ_b values the coherence of qubit a disappears. The effect of WVA for the Bell state is represented in Fig. 2, as well computing the probability of getting such WVA. The associated probability within the WM limit is calculated as $|\langle\psi_i \mid \psi_{post}\rangle|^2$, where $|\psi_i\rangle$ is a Bell state $|\Phi^+\rangle$ and $|\psi_{post}\rangle = |\psi_a\rangle \otimes |\psi_b\rangle$, with $|\psi_a\rangle$ and $|\psi_b\rangle$ being the post selected states for qubits a and b. We observe that although exhibiting an infinite amplification, e.g. when θ_b approaches $2\pi/3$ (blue dashed line), which case represents an unphysical state as the probability for this to happen is zero (see inset in Fig. 2). On the other hand, let us look for a realistic/physical scenario, i.e. when a finite amplification with a non-vanishing probability of success is obtained. Fortunately, in the region where an important amplification takes place, the probability is high enough from an experimental point of view. In Fig. 2 one finds that a twice amplified expectation value, e.g. red dotted line at $\theta_b = \pi/2$, occurs with the probability $\sim 10\%$.

To illustrate more the impact of the control qubit on the WVA we will proceed to measure the initial amount of quantum correlations between the qubits. To advance from simple to more elaborated scenarios, firstly we consider a *Werner* state, i.e. $\rho^Q \equiv \rho_{Werner}$ in Eq.(2). Werner states are a particular case of BD states when $c_1 = c_2 = c_3 = -c$ and they are defined [23] as $\rho_{Werner} = (1-c)\mathbb{I}/4 + c|\Psi^-\rangle\langle\Psi^-|$, where $|\Psi^-\rangle = (|0_a 1_b\rangle - |1_a 0_b\rangle)/\sqrt{2}$.

Furthermore, it is known that Werner states exhibit entanglement if and only if $c \geq 1/3$ (see Fig. 2 in Ref. [23]). Hence, it is clear that the Entanglement of Formation (E) vanishes for $c < 1/3$, while the Quantum Discord (QD) only vanishes at $c = 0$. Following with the result above, we study the role of quantum correlations in the control for the two-qubit case. To achieve this, we show in Fig. 3 the amplification of the weak value given in Eq.(6) for the Werner state ($c_i = -c$). There, we have considered two different projections on the control qubit, $\theta_b = \pi/2$ (blue dashed line) and $\theta_b = \pi/4$ (red dotted line). Without loss of generality, for both cases we have fixed $\phi_b = \phi_a = 0$, $\theta_a = \pi/10$. For $c < 1/3$, we observe the control of WVA with no entanglement, therefore the entanglement does not play a relevant role in setting up the degree of quantum control. Thus, we found in this case that in order to have a control over the target qubit involved in the WVA, one needs to have a resource of quantum correlated states quantified by (in principle) Quantum Discord-like correlation measures rather than non-separability based on, i.e. entanglement.

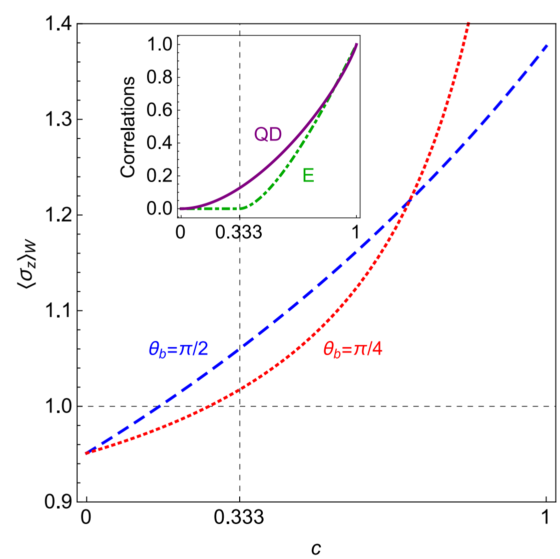

FIG. 3: The weak value $\langle\sigma_z\rangle_W$ in Eq.(6) computed for a *Werner* state can be controlled by the projection of the control qubit b even for zero Entanglement (E) and non-zero Quantum Discord (QD) between the qubits (see Inset). The parameters are $\theta_a = \pi/10$, $\phi_a = \phi_b = 0$ and $\sigma \to \infty$.

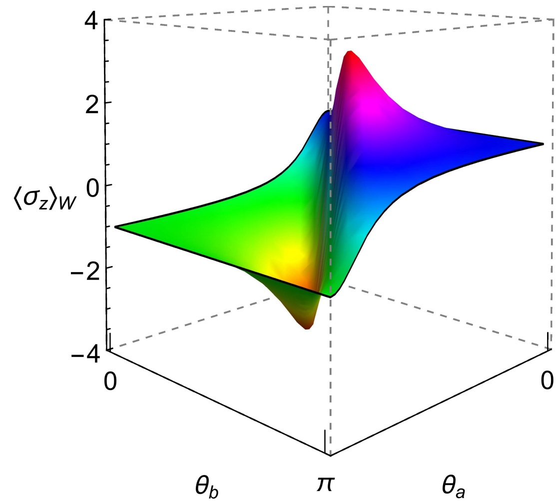

FIG. 4: Weak value for an initial BD state with $\vec{c} = (-0.95, -0.95, -0.9)$ and varying the post-selection states for both qubits, e.g. the angles θ_a and θ_b. Here $\phi_a = \phi_b = \pi/4$ and $\sigma \to \infty$.

For completeness, let us consider the initial uncorrelated state ($QD = 0$), $|\varphi\rangle = (|0\rangle_a + |1\rangle_a)/\sqrt{2} \otimes |0\rangle_b$. For this particular case and following the same procedure as before, it is straightforward to obtain the weak value:

$$\langle\sigma_z\rangle_W = \frac{\cos\theta_a}{1 + e^{-g^2t^2/2\sigma^2}\sin\theta_a\cos\phi_a}. \tag{8}$$

One can see from this equation that the amplification of the mean value is achievable and it is not influenced by the control qubit state, that means that both tracing as well as projecting the quantum state gives the same result. As one would expect, this becomes a clear example of amplification as in AAV of the first qubit that depends on the "weakness of the interaction" with a critical gt value, above which there is no longer amplification, but there is no control from the second qubit, since they are uncorrelated. In fact, one can find that the amplification tends asymptotically to infinity when θ_a approaches $\pi/2$, with $\phi_a = \pi$, although the associated probability goes to zero. To find the "weak" interaction within the weak measurement framework, we proceed to set some routinely values of the quantum dynamics, for instance, $gt = \pi \times 10^{-3}$, and $\sigma = 1/2$ (corresponding to a coherent state). These parameters give a quite accurate approximation of the case $\sigma \to \infty$. On the other hand, when we take larger values of gt, the amplification deteriorates. This suggest an optimal region where the weak interaction takes place. Following a numerical simulation, one could find that $gt \approx 0.3$ corresponds to the threshold where the WVA for the target qubit is achieved for $\sigma = 1/2$, $\phi_a = \pi$ and $\theta_a = 1.4$ rad in Eq. (8).

In Fig.(4) we have depicted the control of the WVA as a function of the pre- and post-selection parameters of both qubits. We have found that, by fixing an initial BD state (i.e. $\vec{c}$) and varying the angles θ_a and θ_b it is possible to optimize the amplification effect. Furthermore, it is straightforward to notice that the behavior of the Entanglement and the QD is similar to the previous Werner case (inset of Fig.3). Therefore, for more general BD states, we have numerically confirmed that the Entanglement plays no role in the control of the WVA. Moreover, we point out on some particular BD states, where the two qubits (target and control) share initially only classical correlations, like states with $\vec{c} = (\pm 1, 0, 0)$ and $\vec{c} = (0, \pm 1, 0)$, which lie on the Cartesian axes [24]. It is interesting to find that for such states with classical correlations, control over the WVA is possible. We present the details of this issue in the Discussion.

Considering the results of this section, we arrive to the following conclusions:

(i) Essentially, the quantum control consists in getting different WVA by manipulating the control qubit through the post-selected angles θ_b and ϕ_b. This is the main result of our paper, since in the original work of AAV [1] —where only one qubit is considered— the amplification depends only on the strength of the weak measurement, say the meter spread σ. In our model, the control qubit b does not interact with the "main" target-meter system and actually it is only connected to the qubit a via the initial correlation. This suggests for the first time the idea that the WVA can be remotely switched on and off.

(ii) Further control might be achieved by choosing conveniently the initial BD state, that is the control via the pre-selection of the two qubits and the quantum or classical correlations between them.

Control of amplification with a squeezed meter state. To introduce another degree of quantum control on the WVA, we will proceed to consider a squeezed meter state. Essentially, we are interested in the ratio gt/σ again, however this time we will relax the Gaussian coherent meter distribution ($\sigma = 1/2$) with a Gaussian squeezed spread one controlled via $\sigma^2 = e^{2r}/4$, being r the squeezing parameter.

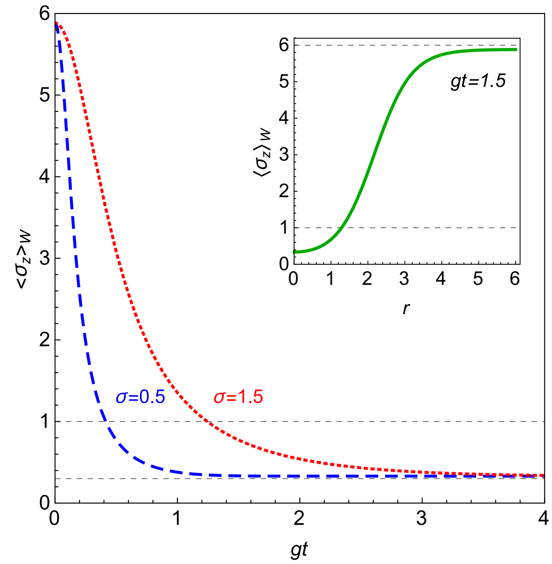

FIG. 5: The weak value for a *Bell* state, i.e. Eq.(7), as function of the dimensionless time and σ. The factor gt sets a threshold for having amplification, which can be moved by tuning σ, i.e. the characteristics width of the meter device. In the inset panel, we consider the case of a squeezed vacuum state, where σ is varied as a function of the squeezed parameter r. Here $\delta = \pi$ and $\theta_a = \theta_b = 1.4$ rad.

For a set of values of $\{\theta_a, \theta_b, \delta\}$ in the main plot of Fig. 5 (blue dashed line), one clearly sees that there is no amplification for the vacuum coherent state ($\sigma = 1/2$) for values higher than $gt_c \approx 0.4$ (c stands for critical value). On the other hand, considering a squeezed vacuum state for the meter, for instance $\sigma = 1.5$ (red dotted line), we are able to push forward this threshold up, e.g. $gt_c \approx 1.2$. The horizontal asymptote valued $\langle\sigma_z\rangle_W \approx 0.3$, corresponds to the case where the interference term (third term in the denominator of Eq.(7) vanishes and one has no amplification for the chosen angles.

In the inset panel of Fig. 5, we show the variation of the weak value as a function of the squeezed parameter r, for $gt = 1.5$. Notice that for $r \lesssim 1.2$ there is no amplification. However, as we increase r further, the amplification starts to appear saturating its value at ~ 6, which is the case when the exponential in Eq.(7) is near to unity.

From Fig. 5 one concludes that in the cases where the exponential term cannot be eliminated, in order to have amplification, the rate gt/σ should be small (weak measurement constraint).

Is multiqubit control more efficient? Three qubits case. For further improvement of our proposal, one may think in adding more qubits for high quantum control. We will show that in fact this is not the case, where the case with only one control qubit is the optimal scheme and that the addition of another one only deteriorates the obtained results. Firstly, a direct generalization of the Bell state, $|\Phi^+\rangle$, that we used along this paper, is the well-known Greenberger-Horne-Zeilinger (GHZ) state,

$$|\Psi\rangle_{GHZ} = (|000\rangle + |111\rangle)/\sqrt{2} \tag{9}$$

Then, one proceeds by fixing the parameters corresponding to the original qubits a (target) and b (first control qubit), say θ_a, θ_b and δ, and taking values that yield amplification with a finite probability. Subsequently, one varies the projection on the third qubit e (second control qubit). For a detailed derivation of the results, see the section Methods. We found numerically, that one can reach higher values for the amplification, but at the expense of having lower probability as the previous one qubit control case. Therefore, such an effect does not lead to any improvement, and even more, it compromises the experimental success.

Nevertheless, in connection to the three qubit scheme, there is more to say about the nature of the quantum correlations involved. It have been proposed and showed that the GHZ state (9) has only genuine three-partite correlations. While a W state, defined as

$$|\Psi\rangle_W = (|100\rangle + |010\rangle + |001\rangle)/\sqrt{3}, \tag{10}$$

has multipartite correlations, e.g. pairwise Entanglement [27]. This means that for the state (9), when tracing out one of the qubits, the two remaining qubits are not quantum correlated. On the other hand, for (10), the opposite happens, and the remaining qubits are maximally quantum correlated. To conclude this section, if one prepares initially the three qubits as GHZ state, when tracing over one or two *control* qubits, the amplification is not possible. However, for the W state, when tracing over only one qubit the amplification persists. This result proves that the control of this type of amplification is intrinsically related to quantum correlations.

Discussion

In the present work, we have studied the quantum control of the weak value amplification (WVA) of a qubit system coupled to a measurement device. On the one hand, a first qubit (target) is directly coupled to the detector device, whereas a second qubit (control) is linked to the former one solely via initial quantum correlations. Motivated by the non-local quantum control of the WVA, we have generalized the single qubit-meter system studied in [1] towards an entangled multiqubits-meter scheme, being the two-qubit correlated scenario the optimal quantum control case. Particularly, our theoretical analysis shows that correlations of purely quantum nature are of pivotal importance for the WVA, i.e., quantum discord rather than entanglement (correlations based on state separability) is the resource that provides the connection and control over the qubit weak value amplification (Figs. 2-5). For instance, in the case of the two qubits being in a $Werner$ state, the quantum control prevails even for the case of zero Entanglement but non-zero Quantum Discord (see Inset of Fig. 3). However, as mentioned previously, our detailed analysis shows that for some cases where the two qubits are initially classically correlated, the control over the WVA could occur. The explanation of these findings is based on the conclusions presented recently by some of us in [26], where it is shown that the presence of coherence in the system is a necessary condition for the existence of WVA, i.e., in our model the measurement of the control qubit b should generate coherence in qubit a. As result, for some BD states with only classical correlations we found that the measurement of the qubit b generates the coherence in the qubit a, so WVA appears; in the case that the coherence is not generated, the WVA is not reported. On the other hand, for the BD states with quantum correlations (QD $\neq 0$), the coherence is always generated in the system as result of the measurement of the control qubit, hence the protocol of WVA control is robust if the QD is present.

Although quantum discord-like or classical correlations are a necessary condition for the generation of the WVA, we require a projective set of individual local quantum operations on each qubit. For instance, for the pre-selected qubits in a general Bell Diagonal (BD) state it is possible to control the WVA via qubit projective post-selection measurements (see Fig. 4).

In the case of achieving WVA, besides the strongly controlled dependence of the amplification due to the phases (θ-azimuthal and ϕ-polar angles on the Bloch sphere) of the post-selected state of the control and target qubits, we have also found that there is a critical gt value for a fixed Gaussian spread of the meter state σ, above which no amplification is fulfilled. This remark is in accordance with the original findings shown in Ref. [1]. There, to gather small amounts of information without perturbing the quantum state, the condition $gt \ll \sigma$ must be attained within the weak measurement framework— as we also require to approximate the unitary evolution operator up to its first order in gt/σ. To illustrate this, we have explored different Gaussian spreads of the meter state by varying its degree of squeezing (see Inset of Fig. 5). We notice that the critical value of gt can be tuned to larger values, as we increase the squeezing parameter σ.

Lastly, as discussed previously, our amplification scheme relies on several quantum control degrees of freedom, being the projective post-selection measurements the most decisive ones to generate qubit WVA. Of course, one may wonder about the feasibility role of the accuracy in the relative qubit phases, as well as the influence of this in the final amplification. To elucidate this, we would like to draw the attention to one particular but powerful application: enhancement of the control qubit measurement accuracy. In other words, we can rely on the weak value amplification protocol to gain further sensitivity on the post-selected phase θ_b. To accomplish this, we make use of the sensitivity given by:

$$\eta = \frac{\langle \sigma_z' \rangle_W - \langle \sigma_z^0 \rangle_W}{\partial/\partial\theta_b \langle \sigma_z' \rangle|_{\theta_b^0}}, \tag{11}$$

where $\langle \sigma_z' \rangle_W$, calculated as in Eq. (7), is the output value measured for a phase θ_b that we assume it is slightly displaced from $\theta_b^0 = \pi/2$. Needless to say that this small deviation is something that one would expect in any realistic experiment and $\langle \sigma_z^0 \rangle_W$ is the theoretical prediction for a perfect measure under ideal conditions. We now proceed to demonstrate that the amplification introduces a higher degree of accuracy. In this direction, we evaluate η for two different phases measured on the target qubit, namely $\phi_a = \pi$ (amplification) and $\phi_a = \pi/2$ (no amplification). We fixed the other parameters to be $\phi_b = 0$, $\theta_a = \pi/3$ and $\sigma \to \infty$. Let us say that our meter, for example, can not detect an angle variation of the output below 1%. Then, when there is no amplification ($\phi_a = \pi/2$), the sensitivity is about 0.01. This means that angles below 0.01 rad can not be resolved in the present configuration. However, when the amplification is switched on ($\phi_a = \pi$) with a non-neglectable probability of $\sim 3.3\%$, the sensitivity corresponds to a value of 0.001. Thus, we can resolve angles up to 0.001 rad, being one order of magnitude smaller than the resolution with no amplification.

We believe that our present work might suggest to develop and/or to implement a new set of experiments and technical tools related to ultra-small signal amplification via remotely controlled weak measurements by one or more correlated qubits.

Methods

Brief theory of weak measurements and weak values. Here we give a summary of the main results of the AAV's standard approach [1]. One begins by preselecting the system S in an initial pure state, $|\psi\rangle$, such that the state of the system is given as $|\psi\rangle = \sum_i \alpha_i |a_i\rangle$, where $\{|a_i\rangle\}$ is the set of eigenstates of the system observable $\hat{\mathbf{A}}|a_i\rangle = a_i|a_i\rangle$.

On the other hand, let $|\phi\rangle$ denote the wave function of the measurement apparatus or device detector modeled in terms of continuous variables $\hat{\mathbf{X}}$ and $\hat{\mathbf{P}}$, such that the initial detector state may be written as $|\phi\rangle = \int \phi(p)dp\,|p\rangle$, with $\phi(p) = (2\pi\sigma^2)^{-1/4}e^{-p^2/4\sigma^2}$, where σ being a measure of the quantum fluctuations. In principle, one could define a WM as the limit when standard deviation σ of the measurement outcome is much larger than the difference between the eigenvalues of the system. For strong measurements, the opposite is true.

The system-detector Hamiltonian, in the interaction picture, can be written as

$$\hat{\mathbf{H}} = g\hat{\mathbf{A}} \otimes \hat{\mathbf{X}}, \tag{12}$$

where g is an interaction constant. Thus, the time evolution operator is $\hat{\mathbf{U}}(t) = \exp\left\{-i\frac{gt}{\hbar}\hat{\mathbf{A}} \otimes \hat{\mathbf{X}}\right\}$, where t is the interaction time. As a result, the global system-detector state after interaction is $|\Psi\rangle = \exp\left\{-i\frac{gt}{\hbar}\hat{\mathbf{A}} \otimes \hat{\mathbf{X}}\right\}|\psi\rangle \otimes |\phi\rangle = \sum_i \alpha_i \int dp\,\phi(p - gta_i)\,|p\rangle \otimes |a_i\rangle$.

If one takes the WM limit and post-selecting the system state $|\psi_{post}\rangle$, the measurement device collapses to the state $|\phi'\rangle = \exp(-i\frac{gt}{\hbar}A_W\hat{\mathbf{X}})|\phi\rangle$, where A_W is the weak measurement value

$$A_W = \frac{\langle\psi_{post}|\hat{\mathbf{A}}|\psi\rangle}{\langle\psi_{post}|\psi\rangle}. \tag{13}$$

and the post-selection success probability is

$$P_{post} = |\langle\psi_{post}|\psi\rangle|^2. \tag{14}$$

For real A_W [16], it is easy to show that $\mid A_W \mid = \frac{\langle\phi'|\hat{\mathbf{P}}|\phi'\rangle}{gt}$, a quantity that in many cases has a value outside the range of the eigenvalues of the observable $\hat{\mathbf{A}}$, in particular in the limit $\langle\psi_{post}|\psi\rangle \longrightarrow 0$. If, in general we write $A_W \equiv A + iB$ as a complex number and let $\mathbf{M}$ be any pointer observable, one can easily prove that

$$\langle\mathbf{M}\rangle_f \;=\; \langle\mathbf{M}\rangle_i + igtA/\hbar\langle\hat{\mathbf{X}}\mathbf{M} - \mathbf{M}\hat{\mathbf{X}}\rangle_i + \frac{gtB}{\hbar}(\langle\hat{\mathbf{X}}\mathbf{M} + \mathbf{M}\hat{\mathbf{X}}\rangle_i - 2\langle\hat{\mathbf{X}}\rangle_i\langle\mathbf{M}\rangle_i), \tag{15}$$

with $\langle\mathbf{M}\rangle_i = \langle\phi|\hat{\mathbf{M}}|\phi\rangle/\langle\phi|\phi\rangle$, $\langle\mathbf{M}\rangle_f = \langle\phi'|\mathbf{M}|\phi'\rangle/\langle\phi'|\phi'\rangle$, where the i and f indices stand for the initial and final (post selection) states.

In particular, if $A_W \equiv iB$ is purely imaginary, then $\langle\mathbf{X}\rangle_f = \langle\mathbf{X}\rangle_i + 2gtB/\hbar Var(\mathbf{X})_i$. On the other hand, when $A_W \equiv A$ is real

$$\langle\mathbf{P}\rangle_f = \langle\mathbf{P}\rangle_i - gtA \tag{16}$$

Solving the integrals.

$K_{10} = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} dp\,(p - gt)e^{-\frac{(p-p_0)^2}{4\sigma^2} - \frac{(p-p_0-2gt)^2}{4\sigma^2}}$, by rearranging the exponential and using the substitution, $\eta = p - p_0$, we get $K_{10} = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{g^2t^2}{2\sigma^2}}\int_{-\infty}^{\infty} d\eta\,(\eta - gt + p_0)e^{-\frac{(\eta-gt)^2}{2\sigma^2}}$.

Now we introduce a second variable $\xi = \eta - gt$, which leads to the result

$$K_{10} = p_0 e^{-\frac{g^2t^2}{2\sigma^2}} \tag{17}$$

The rest of integrals K_{ij} and J_{ij} are calculated similarly.

Weak measurements with many qubits. Let us see what happens if we include a third qubit e in the model, i.e. a second control. We are interested in two different types of tripartite quantum correlated initial states, namely the GHZ in Eq.(9) and W in Eq.(10). We firstly focus on the GHZ initial state and we follow the same procedure as used to derive the numerator and denominator in Eq.(4), but written this time as a function of two control qubits, b and e , which gives

$$tr_M(\rho_{\psi_a}) \;=\; \frac{1}{4}\{2\Pi_{11}^{be}\cos^2(\theta_a/2)J_{11} + 2\Pi_{00}^{be}\sin^2(\theta_a/2)J_{00} + [\Pi_{10}^{be}\sin\theta_a e^{i\phi_a}J_{10} + h.c.]\}, \tag{18}$$

where $\Pi_{ij}^{be} = |ii\rangle\langle jj|$ and J_{ij} were defined previously for Eq.(4). Either if one traces over one qubit and project the other, or trace over both b and e, will not get any amplification. Nevertheless, projecting on both control qubits we found the denominator to be

$$\begin{aligned}\langle\psi_{be}|tr_M(\rho_{\psi_a})|\psi_{be}\rangle \;&=\; \frac{1}{16}\{8\cos^2(\theta_a/2)\cos^2(\theta_b/2)\cos^2(\theta_e/2)J_{11} + 8\sin^2(\theta_a/2)\sin^2(\theta_b/2)\sin^2(\theta_e/2)J_{00}\\ &+\; [\sin\theta_a\sin\theta_b\sin\theta_e e^{i\phi_{abe}}J_{10} + h.c.]\}\end{aligned} \tag{19}$$

where $\phi_{abe} = \phi_a + \phi_b + \phi_e$. One sees that for the weak regime ($\sigma \to \infty$) the solution $\{\theta_a = \theta_b = \theta_e = \pi/2, \phi_a = \phi_b = \phi_e = \pi\}$ leads to amplification (the denominator is zero. However, the strong regime ($\sigma \to 0$) will not yield any amplification, as pointed out in [1] for only one qubit.

For the W initial state Eq.(10) the denominator reads

$$\begin{aligned}tr_M(\rho_{\psi_a}) \;&=\; \frac{1}{6}\{2\Pi_{0000}^{be}\cos^2(\theta_a/2)J_{11} + 2(\Pi_{0101}^{be} + \Pi_{1010}^{be} + \Pi_{0110}^{be} + \Pi_{1001}^{be})\sin^2(\theta_a/2)J_{00}\\ &+\; [(\Pi_{0001}^{be} + \Pi_{0010}^{be})\sin\theta_a e^{i\phi_a}J_{10} + h.c.]\},\end{aligned} \tag{20}$$

with $\Pi_{ijkl}^{be} = |ij\rangle\langle kl|$. Once again, as we found along this work, when tracing over the two control qubits, the amplification is annihilated, since the denominator is $1 + \sin^2(\theta_a/2) \geq 1$. On the other hand, if tracing over b and projecting on e, one gets

$$\langle\psi_e|tr_b[tr_M(\rho_{\psi_a})]|\psi_e\rangle = \tfrac{1}{6}\{2\cos^2(\theta_a/2)\sin^2(\theta_e/2)J_{11} + 2\sin^2(\theta_a/2)J_{00} + \sin\theta_a\sin\theta_e\cos(\phi_a - \phi_e)J_{10}\} \tag{21}$$

The denominator does not vanish, but there is still an interference term: $\sin(\theta_a)\sin(\theta_e)\cos(\phi_a - \phi_e)/2$, which amplifies the expectation value of momentum, i.e. $|\langle p\rangle - p_0|/gt \lessapprox 4$. Therefore, for the initial W state given in Eq.(10), one can still find amplification after tracing over one of the qubits, unlike the GHZ case.

These important differences are related to the quantum correlations, as it is well known for GHZ state after tracing over one of three qubits, all the correlations between them are lost, since the *Quantum Correlations* are purely tripartite. However, for the W state the *Quantum Correlations* remain after tracing over one qubit, which is the reason behind the amplification of the momentum.

[1] Aharonov, Y., Albert D. Z., & Vaidman, L. How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. *Phys. Rev. Lett.* **60**, 1351-1354 (1988).

[2] Rabitz, H., de Vivie-Riedle, R., Motzkus, M. & Kompa, K. Whither the future of controlling quantum phenomena? *Science* **288**, 824-828 (2000).

[3] Wu, S., Mølmer, K. Weak measurements with a qubit meter. *Phys. Lett. A* **374**, 34-39 (2009).

[4] Kofman, A.G., Ashhab, S. and & Nori, F. Nonperturbative theory of weak pre-and post-selected measurements. *Phys. Reports* **520**, 43-133 (2012).

[5] Blok, M. S., Bonato, C., Markham, M. L., Twitchen, D. J., Dobrovitski, V. V. & Hanson, R. Manipulating a qubit through the backaction of sequential partial measurements and real-time feedback. *Nature Phys.* **10**, 189-193 (2014).

[6] Torres, J. P. & Salazar-Serrano, L. J. Weak value amplification: a view from quantum estimation theory that highlights what it is and what isn't. *Sc. Rep.* **6**, 19702 (2016).

[7] Wiseman, H.M. Quantum control: Squinting at quantum systems. *Nature* **470**, 178-179 (2010).

[8] Ashhab, S. and & Nori, F. Control-free control: Manipulating a quantum system using only a limited set of measurements. *Phys. Rev. A* **82**, 062103 (2010).

[9] Wiseman, H.M. & Milburn, G.J. *Quantum Measurement and Control* (Cambridge University Press, 2014).

[10] Palomaki, T.A., Teufel, J.D., Simmonds, R.W. & Lehnert, K.W. Entangling mechanical motion with microwave fields. *Science* **342**, 710-713 (2013).

[11] Basiri-Esfahani, S., Myers, C.R., Combes, J. & Milburn, G.J. Quantum and classical control of single photon states via a mechanical resonator. *New J. Phys.* **18**, 063023 (2016).

[12] von Neumann, J. *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, 1955).
[13] Duck, I.M., Stevenson, P.M., & Sudarshan, E.C.G. The sense in which a "weak measurement" of a spin-1/2 particle's spin component yields a value 100. *Phys. Rev. D* **40**, 2112-2117 (1989).
[14] Vaidman, L. Weak-measurement elements of reality. *Foundations of Physics* **26**, 895-906 (1996).
[15] Vaidman, L. Weak value and weak measurements. *Compendium of Quantum Physics*, Springer(2009), pp.840-842.
[16] Jozsa, R. Complex weak values in quantum measurement. *Phys. Rev. A* **76**, 044103 (2011).
[17] Koike, T., & Tanaka, S. Limits on amplification by Aharonov-Albert-Vaidman weak measurement. *Phys. Rev. A* **84**, 062106 (2007).
[18] Ritchie, N. W. M., Story, J. G. & Hulet, R. G. Realization of a measurement of a "weak value". *Phys. Rev. Lett.* **66**, 1107-1110 (1991).
[19] Pryde, G. J., O'Brien, J. L., White, A. G., Ralph, T. C. & Wiseman, H. M. Measurement of Quantum Weak Values of Photon Polarization. *Phys. Rev. Lett.* **94**, 220405 (2005).
[20] Dixon, P. B., Starling, D. J., Jordan, A. N. & Howell, J. C. Ultrasensitive beam deflection measurement via interferometric weak value amplification. *Phys. Rev. Lett.* **102**, 173601 (2009).
[21] Zhang, L., Datta, A. & Walmsley, I. A. Precision Metrology using weak measurements. *Phys. Rev. Lett.* **114**, 210801 (2015).
[22] Kocsis, S., Braverman, B., Ravets, S., Stevens, M. J., Mirin, R. P., Shalm, L. K. & Steinberg, A. M. Observing the average trajectories of single photons in a two-slit interferometer. *Science* **332**, 1170-1173 (2011).
[23] Luo, S. Quantum discord for two-qubit systems. *Phys. Rev. A* **77**, 042303 (2008).
[24] Lang, M.D. & Caves, C.M. Quantum discord and the geometry of Bell-diagonal states.*Phys. Rev. Lett.* **105**, 150501 (2010).
[25] Liu, B.-H. *et al.*, arXiv:1603.09119 (2016).
[26] Mundarain, D. F. & Orszag, M. Quantumness of the anomalous weak measurement value. *Phys. Rev. A* **93**, 032106 (2016).
[27] Dür, W., Vidal, G. & Cirac, J. I. Three qubits can be entangled in two inequivalent ways. *Phys. Rev. A* **62**, 062314 (2000).

Acknowledgments
M.O. and V.E. acknowledge the financial support of the projects Fondecyt #1140994 and Conicyt-PIA anillo ACT-1112 "Red de analisis estocastico y aplicaciones". R.C. and V.M. acknowledge the financial support of the projects Fondecyt Postdoctorado # 3160154 and # 3160700, respectively.

Author contributions All the authors contributed equally.