Thermal effects on sudden changes and freezing of correlations between remote atoms in a cavity quantum electrodynamics network

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Received February 17, 2014; revised March 31, 2014; accepted April 1, 2014;
posted April 1, 2014 (Doc. ID 206115); published April 23, 2014

We investigate thermal effects on sudden changes and freezing of the quantum and classical correlations of remote qubits in a cavity quantum electrodynamics (CQED) network with losses. We find that the detrimental effect of thermal reservoirs on the freezing of correlations can be compensated via an efficient coupling of the fiber connecting the two cavities of the system. Furthermore, for certain initial conditions, we find a double sudden transition in the dynamics of Bures geometrical quantum discord. The second transition tends to disappear at a critical temperature, hence freezing the discord. Finally, we discuss the feasibility of the experimental realization of the present proposal.

OCIS codes: (270.5585) Quantum information and processing; (270.5580) Quantum electrodynamics.
http://dx.doi.org/10.1364/OL.39.002668

Quantum correlations play a fundamental role in quantum computation and quantum information processing [1], where entanglement is usually considered a popular measure of such correlations. However, more general measures such as quantum discord (QD) [2,3] and geometric quantum discord (GQD) with Bures distance [4–6] have been shown to capture nonclassical correlations, including completely separable systems, e.g., the deterministic quantum computation with one quantum bit (DQC1) model.

The unusual dynamics of classical and quantum decoherence originally reported in [7,8] and confirmed experimentally in [9–11] has stimulated high interest in the investigation of the phenomena of sudden changes in the correlations for different physical systems. Hence, during the past years, intensive effort has been focused on explaining the nature of sudden transitions and freezing effects of quantum correlations and the conditions under which such transitions occur. Also, from the perspective of applications, how efficiently one could engineer these phenomena in quantum technologies is also a matter of interest.

As has been shown in previous studies [6–16], the puzzling peculiarities of the sudden transitions and freezing phenomena are hidden in the structure of the density operator during the entire evolution of a bipartite quantum system for particular decoherence processes. Nevertheless, important questions remain open—how these fascinating effects are affected by the presence of noisy environments and if there are efficient mechanisms to control them in both nondissipative or dissipative decoherence models. The state-of-the-art research of cavity quantum electrodynamics (CQED) networks [17–20] has shown so far modest progress concerning the influence of thermal environments on the correlations, and the sudden transitions and freezing phenomena in particular [13,14,20–27]. This Letter presents interesting novel results in this line of research.

In Fig. 1(b) we show the time evolution of the classical and quantum correlations for a CQED network as discussed in [27] and applied to the case of two excitations, with the qubits initially prepared in a class of states as in Eq. (3) for all the reservoirs at zero temperature. We observe the quantum-classical sudden transition in the case of our CQED model similar to other studied systems [6–16]. Besides the classical correlations (CCs), entropic QD, and relative entropy of entanglement (REE), we have also studied two geometrical measures, geometric entanglement (GE) and GQD defined with Bures distance [4,5]. We notice that the Bures GQD and QD show similar behavior, having flat regions not affected by the dissipation processes during a particular time period, an effect known as freezing of QD. At the same time, CCs decay and meet QD at a point where a sudden change occurs. After this point, the CCs remain constant during another time period until another sudden change follows and so we observe periodic revival of the correlations [9,10,16]. On the other hand, the entanglements show different dynamics, evidencing effects of sudden death and birth not appearing in the QD and GQD for the given system.

In the following we present briefly the model proposed in [26,27], schematically shown in Fig. 1(a), and recall the analytical equations for a generalized model presented in this Letter. Basically, the model considers a quantum open system of two remote qubits (two-level atoms) where each qubit interacts independently with one of

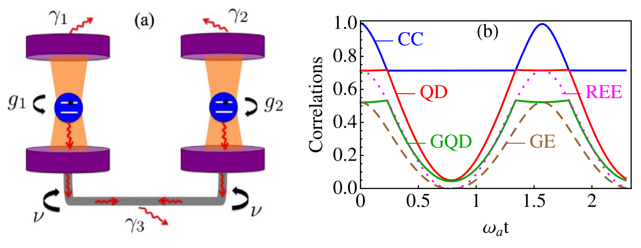

Fig. 1. (a) Remote atoms in the CQED network. (b) Time evolution of the correlations: CC (blue solid), QD (red solid), GQD (green solid), REE (magenta-dotted), and GE (brown-dashed) for the reservoirs at zero temperatures. The parameters are the same as in Fig. 2(a).

the distant cavities coupled by a transmission line (e.g., fiber). For the sake of simplicity, one considers the approach of a short fiber limit: only one active mode of the fiber interacts with the cavity modes [20]. The whole system is open because the cavities and fiber exchange energy with their individual thermal baths; hence, we have a very general case of dissipative decoherence of the quantum system. The Hamiltonian of the system under the rotating-wave approximation in units of $\hbar$ reads

$$H_s = \omega_f a_3^\dagger a_3 + \sum_{j=1}^{2}(\omega_a S_{j,z} + \omega_0 a_j^\dagger a_j) + \sum_{j=1}^{2}(g_j S_j^+ a_j + \nu a_3 a_j^\dagger + \text{H.c.}), \quad (1)$$

where $a_1(a_2)$ and a_3 are the boson operator for cavity 1(2) and the fiber mode, respectively; ω_0, ω_f, and ω_a are the cavity, fiber, and atomic frequencies, respectively; g_j (ν) are the atom (fiber)-cavity coupling constants; and S_z and $S^\pm$ are the atomic inversion and ladder operators, respectively.

One of the important advances and the novelty of the present model compared to previous ones [26,27] is based on the generalization to large number of excitations in the system. To the best of our knowledge, this approach of many excitations in similar systems [17–20] is not common, and may be one of few existing studies.

To describe the evolution of an open quantum-optical system usually the approach of the Kossakowski–Lindblad phenomenological master equation is considered with the system Hamiltonian decomposed on the eigenstates of the field-free subsystems. However, sometimes a CQED system is much more realistically modeled based on the microscopic master equation (MME), developed in [28,29] where the system–reservoir interactions are described by a master equation with the system Hamiltonian mapped on the atom-field eigenstates, known as dressed states. The present system consists of two atoms within their own cavities connected by a fiber. We represent the leakage of the two cavities and the fiber via coupling to individual external environments, thus identifying three independent dissipation channels. Commonly, in CQED the main sources of dissipation originate from the leakage of the cavity photons due to the imperfect reflectivity of the cavity mirrors. Another mechanism of dissipation corresponds to the spontaneous emission of photons by the atom; however, this kind of loss is negligibly small in the CQED regime considered in our model and, consequently, is neglected. Hence, it is straightforward to bring the Hamiltonian H_s in Eq. (1) to a matrix representation in the atom-field eigenstates basis. To define a general state of the whole system we use the notation $|i\rangle = |A_1\rangle \otimes |A_2\rangle \otimes |C_1\rangle \otimes |C_2\rangle \otimes |F\rangle \equiv |A_1A_2C_1C_2F\rangle$, where $A_{1,2}$ correspond to the atomic states, which can be $e(g)$ the for excited (ground) state, while $C_{1,2}$ and F define the cavities and fiber states, respectively, which may correspond to 0, 1, … n photon states. Because the quantum system is dissipative, the excitations may leak to the reservoir degrees of freedom, hence the ground state of the system, $|0\rangle = |gg000\rangle$, should be also considered in the basis of the states. Therefore, in the case of N excitations in our system, the number of dressed states, $|i\rangle$, having a minimum of one excitation, i.e., excluding the ground state $|0\rangle$, is computed by a simple relation: $d_N = N + 2\sum_{k=1}^{N} k(k+1)$. For example, in the case of $N = 2$ excitations the Hamiltonian H_s in Eq. (1) is decomposed in a state-basis of dimension $1 + d_2$, i.e., it is a 19×19 matrix; for six excitations, H_s is represented by a 231×231 matrix, and so on. Hence it is evident that for large N the general problem becomes hard to solve even numerically. In the present work, we develop our calculations up to six excitations, which is an improvement as compared to some previous works with, e.g., two excitations [20].

Considering the above assumptions and following the approach of [28,29], the MME for the reduced density operator $\rho(t)$ of the system is defined in the form given by Eq. (2) in [27]. In the following we develop the equation for the density operator $\rho(t)$ mapped on the eigenstates basis, $\langle\phi_m|\rho(t)|\phi_n\rangle = \rho_{mn}$ for the case of N excitations in the system:

$$\dot{\rho}_{mn} = -i\bar{\omega}_{n,m}\rho_{mn} + \sum_{k=1}^{d_N}\left[\frac{\gamma_{k\to 0}}{2}(2\delta_{m0}\delta_{0n}\rho_{kk} - \delta_{mk}\rho_{kn} - \delta_{kn}\rho_{mk}) + \frac{\gamma_{0\to k}}{2}(2\delta_{mk}\delta_{kn}\rho_{00} - \delta_{m0}\rho_{0n} - \delta_{0n}\rho_{m0})\right], \quad (2)$$

where δ_{mn} is the Kronecker delta; the physical meaning of the damping coefficients $\gamma_{k\to 0}$ and $\gamma_{0\to k}$ refers to the rates of the transitions between the eigenfrequencies Ω_k and Ω_0 downward and upward, respectively, defined as follows: $\gamma_{k\to 0} = \sum_{j=1}^{3} c_i^2 \gamma_j(\bar{\omega}_{0,k})[\langle n(\bar{\omega}_{0,k})\rangle_{T_j} + 1]$, and by the Kubo–Martin–Schwinger condition we have $\gamma_j(-\bar{\omega}) = \exp(-\bar{\omega}/T_j)\gamma_j(\bar{\omega})$, where c_i are the elements of the transformation matrix from the states $\{|0\rangle, |1\rangle, \ldots, |d_N\rangle\}$ to the states $\{|\phi_0\rangle, |\phi_1\rangle, \ldots, |\phi_{d_N}\rangle\}$ {similar to Eq. (14) and Appendix A in [26]}. Here $\langle n(\bar{\omega}_{\alpha,\beta})\rangle_{T_j} = (\mathrm{e}^{(\Omega_\beta - \Omega_\alpha)/T_j} - 1)^{-1}$ corresponds to the average number of thermal photons (with $k_B = 1$). The damping coefficients play a very important role in our model because their dependence on the reservoir temperatures imply a complex exchange mechanism between the elements of the system and the baths. Further, one solves numerically the coupled system of the first-order differential equations [Eq. (2)] and compute the evolution of different kinds of correlations between the two distant atoms, given some finite temperature of the reservoirs. In order to get the reduced density matrix for the atoms, one performs a measurement on the cavities and the fiber vacuum states, $|000\rangle = |0\rangle_{C1} \otimes |0\rangle_{C2} \otimes |0\rangle_F$; later we will explain how this task can be realized experimentally. We find that, after the projection, the reduced atomic density matrix has a X form and the correlations can be computed easily as developed in [4,30–32].

The system under consideration refers to atoms with long radiative lifetimes trapped in their own cavities and connected by a fiber. The cavities and fiber exchange their energy with individual reservoirs [Fig. 1(a)] which, for the sake of simplicity, are taken to have the same damping rate $\gamma_1 = \gamma_2 = \gamma_3 \equiv \gamma$. The transition frequency of the atom is considered a free parameter used to scale the rest of the dimensionless parameters, which we take as being similar to some experimental data [33], e.g., $\omega_a/2\pi = 10$ GHz. The atom–cavity couplings satisfy the constraint of the MME in a Markovian environment, i.e., $2g \gg \gamma$ [29] and we set the values $g_1 = g_2 = g = 10\gamma$ in Figs. 1 and 2. The values of γ and ν will be tuned to evidence the effects of the thermal baths. We find that the detunings do not have an important impact on the effects we discuss here, so we set the values $\omega_f = \omega_a$ and $\omega_a - \omega_0 = 0.1\omega_a$. In the following, we compute the time evolution of the atomic correlations, keeping in mind the main objective of this Letter, which is to find the influence of thermal baths on these correlations. To compute the general correlations—classical and quantum for the given system—we consider the concepts of mutual information, classical correlations, and entropic QD as defined and calculated in [2,3,30–32], as well the GQD with Bures distance, recently developed by one of us [4,5] and independently in [6]. Let us consider that the two atoms initially are prepared in a particular state as Bell-diagonal (BD), described by an X-type density matrix in Bloch form as

$$\rho(0) = [I \otimes I + \vec{c} \cdot (\vec{\sigma} \otimes \vec{\sigma})]/4, \tag{3}$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the vector given by Pauli matrices, I is the identity matrix, and the vector $\vec{c} = (c_1, c_2, c_3)$ defines completely the state with $-1 \le c_i \le 1$. It is very important to point out here that the majority of the works studying the sudden transition and freezing effects of the correlations in the bipartite quantum system consider the special decoherence mechanisms (noise channels known as bit flip, phase flip, and bit-phase flip [6,8,34]) so that during the time evolution the density matrix preserves the property of its initial state, i.e., $\rho_{11}(t) = \rho_{44}(t) = [1 + c_3(t)]/4$ and $\rho_{22}(t) = \rho_{33}(t) = [1 - c_3(t)]/4$. For this scenario the classical correlations and QD are easily computed with the help of the work of Luo in [30] or given explicitly in [8]. However, for quantum systems embedded in natural environments, like heat baths, the above-mentioned equality of the density matrix elements is no longer satisfied, as the system evolves in time, even

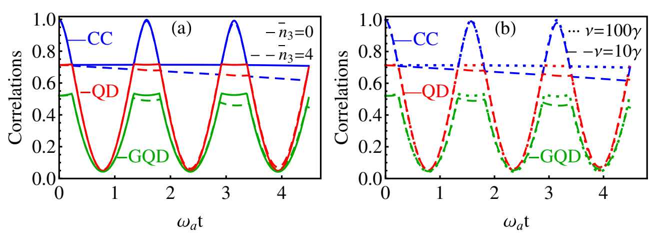

Fig. 2. Dynamics of the correlations: CC (blue), QD (red), and Bures GQD (green) for $\gamma = 0.008\omega_a$. (a) Varying the temperature of the fiber's reservoir given by the average number of the thermal photons, i.e., $\bar{n}_3 = 0$ (solid line) and $\bar{n}_3 = 4$ (dashed line) for constant cavity–fiber coupling $\nu = 10\gamma$. (b) Varying the cavity-fiber coupling, $\nu = 10\gamma$ (dashed line) and $\nu = 100\gamma$ (dotted line) for constant $\bar{n}_3 = 4$. The initial state is defined by $\vec{c} = (1, -0.9, 0.9)$ in Eq. (3).

if initially we have a BD state. Hence, it is of great interest to study the phenomena of sudden transition and freezing of the correlations for more general (realistic) dissipation models, such as our system under the MME approach.

Recently, Pinto *et al.* in [12] discussed the sensitivity of the sudden change of the QD to different initial conditions. In this context, the present work shed more light on this important subject. In particular, for the system under study, we find that the atomic density matrix preserves the initial BD form only for a short time under the action of the heat reservoirs. It very quickly evolves into a more general X-shaped non-BD form. Under these circumstances, we compute the QD as shown in Figs. 2 and 3 by using a more general algorithm developed in [31,32]. Furthermore, we also make use of an alternative, nonentropic measure of the quantum correlations, such as the GQD with Bures distance, proposed and calculated for a bipartite system in [4,5]. Hence, observing in Fig. 1(b) the effect of the sudden change and freezing of the correlations for the reservoirs at zero temperature, it is natural to inquire about the thermal effects on the classical and quantum correlations when the cavities and fiber are connected to reservoirs at finite (nonzero) temperatures. In our numerical analysis, we find that the freezing effects of the QD and GQD decay by increasing individually or collectively the temperatures of the cavities or the fiber. In Fig. 2(a) we show the effect of heating the fiber to four thermal photons and observe that the thermal effects act destructively on the freezing of both the entropic and geometric discords. However, the sudden transitions persist. The next question is: Could one recover from the damaging effects of the system being coupled to the thermal reservoirs? We find that we could, in principle, engineer such a recovery by a suitable increase in the fiber–cavity coupling. As a matter of fact, we show in Fig. 2(b) that, when we set the fiber's bath temperature to three thermal excitations, such recovery of the correlations, via the increase of the fiber–cavity coupling, is feasible, hence by this effect we understand the important role of the photon as the carrier of the quantum correlations between the remote qubits in such a network.

Recently in [34] the authors theoretically described another interesting class of sudden transitions and freezing of the quantum correlations, which later was observed experimentally in NMR setups [35]. They found the formation of an environment induced double transition of Schatten one-norm geometric quantum correlations

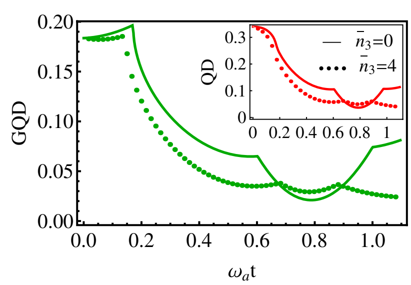

Fig. 3. Dynamics of GQD evidencing double sudden transitions (QD in inset). The parameters considered here are $\gamma = 0.1\omega_a$, $\nu = g = 5\gamma$ for $\bar{n}_3 = 0$ (solid line), and $\bar{n}_3 = 4$ (dotted line). The initial state is given by $\vec{c} = (0.85, -0.6, 0.36)$.

(GQD-1), which is not observed in the classical correlations, and is thus a truly quantum effect.

Motivated by this very recent finding, we simulate the dynamics of the Bures GQD for our model and find a type of double sudden transition somewhat different from the ones observed in [34,35]. To the best of our knowledge, this kind of double sudden transition and freezing effect for Bures GQD has not been reported in literature and by this result we come to an important conclusion that both the GQD-1 and Bures GQD show similar quantum effects. In Fig. 3 we see the double sudden changes in the dynamics of Bures GQD for the reservoirs at zero temperatures, while the QD suffers one sudden change. We observe the following interesting result: as we increase the temperature of the fiber's bath, there is a peculiar tendency to freeze the GQD and the second transition tends to disappear at a critical temperature.

The experimental realization of the present proposal hinges on the possibility of realizing quantum nondemolition (QND) measurements of the photon states in the fiber–cavities system. There is extensive literature on QND measurements in CQED; for a review see [33]. In our scheme we propose to measure the two-qubit density matrix in the condition that all the fields are in vacuum state, so it is feasible to monitor the probability of this state during the temporal evolution of the system [27].

In summary, we analyzed here the phenomena of sudden changes and freezing of correlations in a CQED network with thermal dissipation channels (Fig. 1). Double sudden transitions for Bures GQD are observed for the first time, to the best of our knowledge (Fig. 3). We conclude that by controlling the dissipation mechanisms one may engineer the quantum correlations with multiple sudden changes and freezing periods in the temporal evolution, effects which may have practical applications. A kind of thermal critical effect in this model is expected, as it is in other systems [36].

We acknowledge fruitful discussions with V. Montenegro, D. Spehner, and L. Morales-Molina. This work is supported by Chilean Fondecyt project no. 100039, and Programa de Investigacion Asociativa anillo ACT-1112.

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