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Model based on a quantum algorithm to study the evolution of an epidemics

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Abstract

A model based on a quantum algorithm is used to study the spread of HIV virus and to predict infection rates on individuals who are not aware of their particular condition. The model makes an analogy between quantum systems and individuals who are infected by the disease. Individuals are represented by two-level quantum systems (quantum "bit"), and the interactions among individuals who cause the infection are represented by unitary transformations. The population is divided into categories according to their behaviour, and the interactions among those individuals in the same category and those in different categories are simulated. The objective is to obtain statistical data on the number of infected individuals depending on the time for every category and for the entire population. Besides, we analyse the impact of the evolution of the disease on individuals who have not knowledge of their specific sanitary condition.

Keywords: Quantum computation; Epidemic; HIV dynamics

0. Introduction

There is a substantial number of studies on epidemics which use various types of models [1,2]. Kermack and McKendrick [3], in a pioneering work on epidemiology, and using a simple model to study the evolution of an epidemics, showed that their model was well-fitted to analyse data collected from the Bombay plague in 1906. Models to study AIDS are discussed by Murray [1] and May and Anderson [4]. An excellent review on the Kermack–McKendrick papers was written by Anderson [5].

The purpose of this paper is to study the evolution of HIV using a model based on a quantum algorithm. We consider a population of N individuals who change in time. Each individual is represented by a qubit (two-level quantum system). A qubit level represents an infection state, either positive or negative. If an individual does not know his particular infection condition, he will be represented by a qubit that indicates a superposition of levels 0 and 1. The amplitude of probability for each level depends on his/her previous behaviour under situations of risk.

The evolution of the system is described by unitary transformations between qubits. These transformations represent potential forms of infection among individuals. For example, a particular sexual intercourse will be represented by a quantum logical gate with two entrances (interacting qubits), and the exit to this gate corresponds to the state of each qubit after the intercourse. The state of each individual will change according to the interactions with the other qubits.

We can measure the state of a qubit at any given moment. When a quantum system that is in a superposition of two states is measured, the result will be one of two states. The system will remain in that state after the measurement. The quantum system can be in a superposition state while the measurement is not made [6–9].

1. Mathematical formulation of the model

A bit is the fundamental concept of classical computing and classical information. Quantum computing and quantum information are built upon a similar concept, the quantum bit, or qubit. A classical bit has a state (0 or 1), while a qubit [10] can have two possible states $|0\rangle$ and $|1\rangle$, which correspond to the states for a bit. Notation such as '$|\rangle$' is called Dirac notation, which will be used extensively throughout this paper since it is the standard notation to represent states in quantum mechanics.

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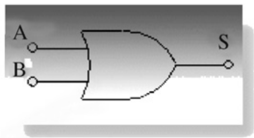

A	B	A or B
0	0	0
0	1	1
1	0	1
1	1	1

Fig. 1. Classical gate OR.

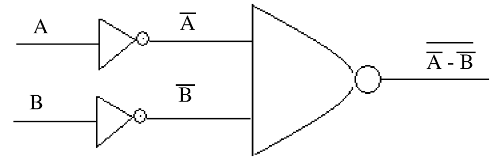

Fig. 2. Equivalence between gate OR and gate NAND.

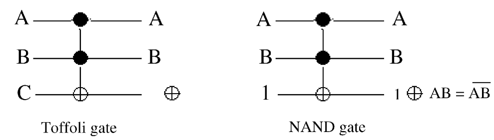

Fig. 3. The Toffoli gate.

The difference between bits and qubits is that a qubit can be in a state $|0\rangle$ or $|1\rangle$, and it can also form linear combinations of states, often called superpositions:

$$|\Psi\rangle = a|0\rangle + b|1\rangle. \tag{1}$$

Numbers a and b are complex numbers, although they can be considered real numbers. In other words, the state of a qubit is a vector in a complex, two-dimension vector space. The special states $|0\rangle$ and $|1\rangle$ are known as computational basis states and form and orthonormal basis for this vector space.

While a bit can be examined to determine whether it is in a 0 or 1 state—for example, when computers retrieve data from RAM—a qubit cannot be examined to determine its quantum state, that is, the a and b states. Instead, in quantum mechanics we can only acquire restricted information about the quantum state. When a qubit is measured we get either 0, with $|a|^2$ probability, or we get 1, with $|b|^2$ probability. Naturally, $|a|^2 + |b|^2 = 1$, since probabilities must be one. Geometrically, this means that the qubit's state is normalised to length 1.

The unobservable state of a qubit and the observations that can be possibly made lies at the heart of quantum computing and quantum information [10]. In most abstract models, there is a direct correspondence between abstract elements and the real world. For example, the plans for designing a building correspond more or less with the final construction. There is not a direct correspondence of this sort in quantum systems. Thus, it is difficult to infer their behaviour. However, there is an indirect correspondence because qubit states can be manipulated and transformed so that the results of their measurements depend on the different properties of their distinctive states [10].

Taking this dichotomy into account, it is possible to make an analogy to the individuals affected by an epidemics in a given population. Every individual in this population is associated to a qubit $|\Psi\rangle = a|0\rangle + b|1\rangle$. State $|0\rangle$ corresponds to a non-infected individual and state $|1\rangle$ corresponds to an infected individual. For the individual represented with qubit $|\Psi\rangle$, the probability of infection is $|b|^2$. For the interaction among the elements of the population (qubit), we will use the logical gate "or". Fig. 1 shows the symbol for this gate and the table of entries and exit.

If state "0" corresponds to the non-infected condition and state "1" to the infected condition, then "A" or "B" represent the condition of infection for A and B after an interaction (e.g. sexual intercourse). This gate is defined by classical bits and does not have a quantum analogy due to the fact that classical gates are not usually reversible, whereas logical quantum gates are reversible [11–14]. Therefore, we must find a quantum circuit that may allow us to carry out the same function of gate OR.

Fig. 2 shows a gate used to carry out the function of the classical gate OR in studying interactions among individuals who do not know if they are either in "0" or "1" state. Therefore, they are in a linear combination of these two states.

There are two "NOT" gates and one "NAND" gate. States 0 and 1 are interchanged within the "NOT" gate.

The NOT quantum gate acts linearly, i.e. it takes the following state:

$$a|0\rangle + b|1\rangle. \tag{2}$$

Then, it takes the corresponding state in which the roles of $|0\rangle$, $|1\rangle$ are interchanged:

$$a|1\rangle + b|0\rangle. \tag{3}$$

A quantum gate can be represented as a matrix, which is directly derived from the linearity of quantum gates. An X matrix is defined to represent the NOT quantum gate as follows:

$$X \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{4}$$

If the quantum state $a|0\rangle + b|1\rangle$ is written in vector notation as

$$\begin{bmatrix} a \\ b \end{bmatrix} \tag{5}$$

with the top entry corresponding to the amplitude for $|0\rangle$ and the bottom entry to the amplitude for $|1\rangle$, then the corresponding output from the NOT quantum gate is

$$X|\Psi\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}. \tag{6}$$

Any classical circuit can be replaced by an equivalent circuit containing only reversible elements using a reversible gate known as the Toffoli gate. The Toffoli gate has three input bits and three output bits, as shown in Fig. 3. Two of these bits are control bits that are unaffected by the action of the Toffoli gate. The third bit is a target bit that is flipped if both control bits are set to 1. Otherwise, it is left alone. Note that applying the Toffoli gate twice to a set of bits has the effect $(a, b, c) \to (a, b, c \oplus ab) \to (a, b, c)$. Thus, the Toffoli gate is inherently reversible. The $\oplus$ operation corresponds to the classical gate "XOR". The Toffoli gate simulates the NAND gate if the third bit is set to the standard state 1, as shown in Fig. 3.

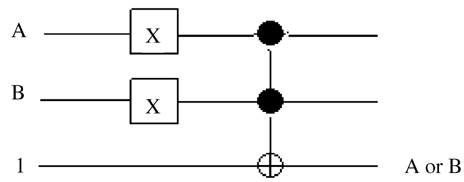

Fig. 4. Quantum circuit that simulates A or B.

The Toffoli gate has been described as a classical gate, but it can be implemented as a logical quantum gate as well. By definition, the logical quantum implementation of the Toffoli gate permutes computational basis states in the same way as the classical Toffoli gate. For example, a Toffoli quantum gate acting in state $|1\,1\,0\rangle$ flips the third qubit because the first two are already set, thus resulting in state $|1\,1\,1\rangle$. It is possible to write this transformation as an 8 by 8 matrix, U, and verify that U is an unitary matrix. Thus, the Toffoli gate is a legitimate quantum gate. As for the amplitudes for $|0\,0\,0\rangle$, $|0\,0\,1\rangle$, $|0\,1\,0\rangle$, $|0\,1\,1\rangle$, $|1\,0\,0\rangle$, $|1\,0\,1\rangle$, $|1\,1\,0\rangle$, and $|1\,1\,1\rangle$, the U matrix may be written as

$$U=\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}. \tag{7}$$

In this way, the quantum circuit simulating the interaction between two individuals (A) and (B) in the target population is shown in Fig. 4.

2. Results

The model has been validated analysing the spread of HIV in Chile between 1986 and 2000. The methodology used was the following.

An equal number of qubits to the population of the country in every year is considered. The initial condition (1986) is 12 million qubits, of which 600 are in state $|\Psi\rangle = a|0\rangle + b|1\rangle$ with $|b|^2 = 0,49$. This means that each individual has a 49% probability of being infected. The remaining population is in quantum state $|0\rangle$.

The dynamics of the infection is simulated by dividing 1 year into 52 weeks. Sixty-three percent of the population represents sexually active individuals [15,16]. In each week, two members of this segment are randomly chosen to apply the operation described in Fig. 4. Vertical infection (mother to son) cases, amounting to 1.5% of the total number of cases in Chile, are not considered in this simulation.

The process is repeated N_0 every week. At the end of every year, 5% of the total population is chosen at random in order to evaluate their quantum state. As previously said, when a quantum system is measured —when states are superposed—we can only obtain one of two possible states.

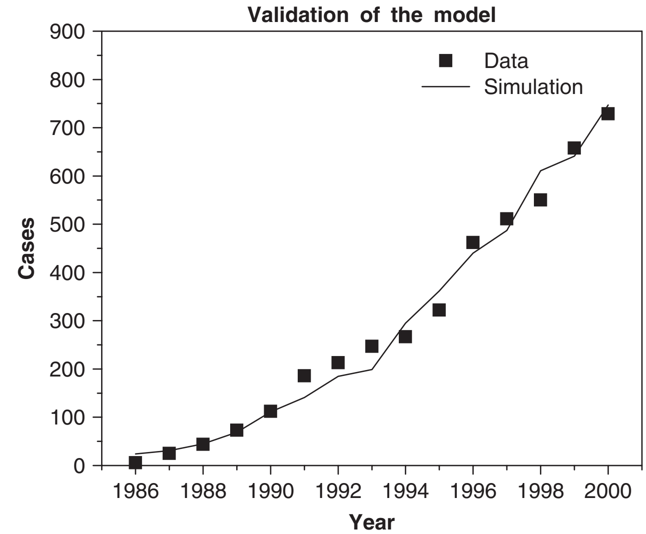

Fig. 5. Number of cases, Chile, 1986–2000.

Demographic data show that Chilean population increases every year. Incorporated individuals enter the circuit in state $|0\rangle$.

The value of parameter N_0 decreases automatically in time. This implies that since the disease was known in Chile, risky interactions between individuals began to decrease.

Fig. 5 shows the behaviour of the model. It shows real data [15] and the data obtained with the model.

Fig. 6 shows the number of reported cases [15,16] and the number of cases given by the model, as well as incidence rates.

An important result that can be achieved with the model is the projection of the total number of HIV-positive individuals. The methodology that has been used in Chile is consistent with the one used in the rest of Latin America and The Caribbean [17]. The method consists of multiplying the current number of HIV cases by 3.76, and then the number of individuals infected by HIV is added. However, in the model discussed in this paper, a measurement of all qubits composing the population is made at the end of every year. When we compare the results obtained with both methods, we can find significant differences. The present model can predict a larger number of infected individuals compared to the official predictions. A comparison of results can be made in Fig. 7 and 8.

3. Summary

The present model shows a sound correlation to actual data on infection by HIV in Chile during the specified period of time (1986–2000). The adjustment parameters are: (1) the N_0 number, which corresponds to the number of times in which individuals interact per week; (2) the percentage of individuals chosen every year in order to measure their state; and (3) the initial state with the number of infected individuals. In Fig. 8, we observe a leap between 1998 and 1999, which is explained by parameter N_0, which changes in a discreet way (on average,

Table 1. Number of cases and rate of asymptomatic infection by HIV per year of diagnosis. Chile 1986 - 2000

Year	Cases	Cases (model)	Incidence Rates	Incidence Rates (model)
1986	6	24	0.05	1.20
1987	25	31	0.20	0.25
1988	44	45	0.35	0.36
1989	73	69	0.57	0.54
1990	112	111	0.85	0.84
1991	186	141	1.40	1.06
1992	213	185	1.57	1.36
1993	247	199	1.79	1.44
1994	267	295	1.91	2.11
1995	322	362	2.27	2.55
1996	462	440	3.20	3.05
1997	511	487	3.49	3.33
1998	550	611	3.71	4.12
1999	658	641	4.38	4.27
2000	729	747	4.79	4.91

Rates per 100,000 inhabitants.

Fig. 6. Number of HIV-infected individuals, Chile, 1986–2000.

Table 2. Projection of infected people with asymptomatic HIV

Year	Infected(data)	Infected (model)
1986	29	572
1987	119	457
1988	209	777
1989	347	803
1990	533	1435
1991	885	2090
1992	1014	1943
1993	1176	1964
1994	1271	2875
1995	1532	3423
1996	2199	3718
1997	2432	5589
1998	2618	5904
1999	3232	4028
2000	3470	4115

Fig. 7. Projection of infected people with asymptomatic HIV.

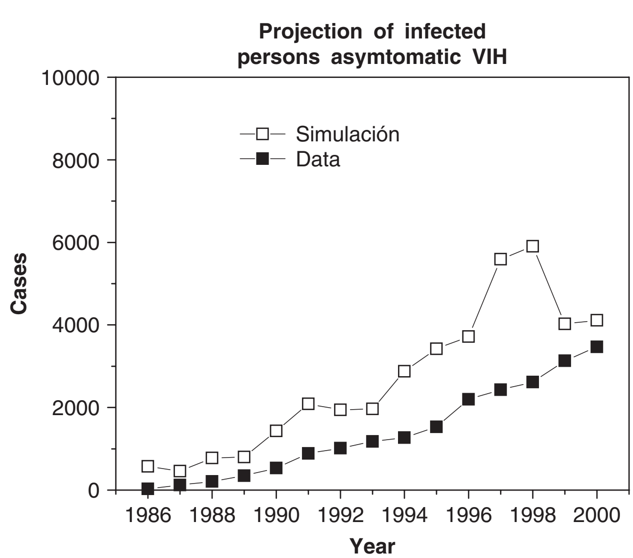

Fig. 8. Total number of infected people.

every 3 years). The leap is more important during this period. From a biological point of view, the situation can be interpreted as the consequence of sanitary measurements taken by Chilean people 5 years before that period, when the disease was already well-known by the population. The model presented in this paper can have great potential to be applied in specific situations due to the fact that it can simulate an examination of the complete population under study. We intend to open a discussion in Latin American countries on the different methods that are used to estimate the total number of HIV-infected individuals.

Further applications of the model can be in the field of infection at the cellular level, including different pathogenic agents. For these applications, the quantum gates that shape the interaction must be modified accordingly. The development and evolution of different epidemic diseases must be studied as well.

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