Soliton instabilities in the easy plane ferromagnet Heisenberg chain with out-of-plane spin deviation

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In this paper we investigate the presence of out-of-plane spin deviation $\rho(\xi)$ in the easy-plane ferromagnetic Heisenberg chain by using the coupled-boson operators together with the Schwinger transformation for the spin operator; this method allows us to conclude that the critical behaviour of the instability is due to the velocity of the nonlinear excitations (solitons) only for an appropriate range of the magnetic field. In this case, when the velocity becomes lower, the stable soliton corresponding to $\rho(\xi)$ is distorted by magnons and loses stability. If we increase the velocity of $\rho(\xi)$, it then decays into high frequency-oscillations. Nevertheless, we find an opposite competence effect produced by the velocity and the magnetic field on $\rho(\xi)$.

Keywords: Ferromagnetic Heisenberg chain; solitons; easy plane; out-of-plane spin deviation.

En este trabajo se investiga la presencia de las desviaciones del espín fuera del plano $\rho(\xi)$, en el plano de fácil magnetización de una cadena ferromagnética de Heisenberg, utlizando operadores bosónicos acoplados en conjunto con las transformaciones de Schwinger para los operadores de espín, este método permite concluir que el comportamiento crítico de las inestabilidades, se debe a la velocidad de las excitaciones no lineales (solitones). En este caso, cuando la velocidad llega a ser baja, el soliton estable correspondiente a $\rho(\xi)$ es distorsionado por magnones y pierde estabilidad. Si se incrementa la velocidad de $\rho(\xi)$ entonces este decae en oscilaciones de alta frecuencia. No obstante lo anterior, se encuentra un efecto de competencia opuesto entre los efectos producidos por la velocidad y el campo magnético sobre $\rho(\xi)$.

Descriptores: Cadena ferromagnética de Heisenberg; solitones; plano de fácil magnetización; desviaciones del espín fuera del plano.

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1. Introduction

Model substances representing magnetic chains have been investigated in detail by measuring neutron scattering cross sections Boucher [1] and by several other experimental techniques, Seit and Berner [2]. The results reveal the presence of nonlinear excitations of the freedom's magnetic degrees, which are as elementary as the usual linear modes. A particular system which theories and experiments have intensively been studied is the easy-plane linear-chain magnet $CsNiF_3$, in which the interactions between the spins are ferromagnetic; to compare theory and experiments, many approximations have been produced, in particular those involving a sine-Gordon mapping Mikeska [3].

The easy-plane magnetic behaviour of this system can be well represented by an easy-plane Heisenberg Hamiltonian from which it is fairly straightforward used to obtain nonlinear excitations (solitons, domain-walls) if one assumes some simplifying characteristics present in the low temperature regime; for example, a classical approach for the spin variables makes it possible in the case of extreme anisotropy, *i.e.* $(D/J)^{1/2} \gg k_B T$) and in the continuum limit, to map the Hamiltonian to a sine-Gordon equation whose static and dynamic properties are well known. However, there are lively discussions about the importance of the out-of-plane spin deviation in interpreting experimental data. Moreover, quantum contributions to the statistical behaviour of this system are also expected due to the fact the spins are quantum variables associated with a discrete one-dimensional regular lattice.

Since the prediction of solitary spin structures in the planar ferromagnet [3], the main effort was focused on the stability of these spin structures, according to Magyari and Thomas [4]. The kinks in a planar ferromagnet with an in-plane magnetic field are unstable above a critical-field; this critical field strength decreases rapidly as the kink velocity increases toward its maximum value (likewise Kumar, [5]). An analysis of the limitations of the sine-Gordon (s–G)-like description of an easy-plane ferromagnetic chain is reported, and shows that the (s–G) description is valid only for low magnetic fields and low velocities of soliton motion (also Mikeska and Osano [6]). They present results of the nonlinear dynamics of a realistic classical easy-plane ferromagnetic chain in an external magnetic field, in particular concerning the dependence of soliton solutions on the strength of the single-ion anisotropy, taking into account nontrivial aspects of the dynamics of the spin S^z (the out-of-plane component) (see, in the same regard, Seit and Bernner [2]). By means of the nuclear spin-lattice relaxation of Cs^{133}, they study linear and nonlinear magnetic excitations in the one-dimensional easy-plane ferromagnet ($CsNiF_3$). It is shown that the effect of out-plane fluctuations remains an open question.

Recently, interest in the study and comprehension of these systems still remains valid. Research into the nonlinear properties of magnetic systems have attracted a great deal of attention due to the new experimental data and the possibility of their wide applications to different branches of applied science and technology [7,18].

In this paper, our purpose is to investigate the existence and the behaviour of the out-of-plane spin deviation in the ferromagnetic Heisenberg chain with easy-plane anisotropy in the presence of an external magnetic field H parallel to that plane.

2. Model and theory

The model considered in the present paper is used to take into account the effects produced by the out-of-plane deviations in the magnetic system, where the Hamiltonian for the magnetic chain is given by:

$$\mathcal{H} = -\frac{J}{2}\sum_{n,\delta=\pm a}^{N} \vec{S}_n\cdot\vec{S}_{n+\delta} + D\sum_{n=1}^{N}(S_n^z)^2 - h\sum_{n=1}^{N} S_n^x, \quad (1)$$

where $J>0$ is the exchange nearest-neighbour interaction, $\vec{S}_n$ the spin on site n, $D>0$ represents the intensity of the local anisotropy, and $h\equiv g\mu_B H$.

Proceeding as Ferrer and Pozo do [19], we take the continuum form of the Hamiltonian:

$$\begin{aligned}\mathcal{H} = &-\frac{J}{2}\int dz\left(\frac{1}{2}(S^\dagger(z)S^{-\prime\prime}(z) + S^-(z)S^{\dagger\prime\prime}(z))\right.\\ &\left.+\, S^z(z)S^{z\prime\prime}(z)\right) + D\int dz(S^x(z))^2\\ &- h\int dz S^x(z) - \frac{1}{2}NJS^2. \end{aligned}\quad (2)$$

Following Ferrer and Pozo [20], making use of the Schwinger coupled-boson representation for the spin operators, we can write:

$$S^\dagger(z) = a^\dagger(z)b(z); \quad S^-(z) = b^\dagger(z)a(z) \quad (3)$$
$$S^z(z) = \frac{1}{2}[a^\dagger(z)a(z) - b^\dagger(z)b(z)],$$

where $a^\dagger(z), a(z), b^\dagger(z), b(z)$ are the usual simple harmonic oscillator bosonic operators acting at each point of the chain.

In the Heisenberg picture, the dynamic of the above operators is coupled through equations ($\hbar=1$)

$$i\dot{a}(z,t) = [a(z,t),\mathcal{H}]; \quad i\dot{b}(z,t) = [b(z,t),\mathcal{H}]. \quad (4)$$

From these equations, we find two non-linear differential equations for the spin operators; then, we take the classical limit of the spin variable, obtaining:

$$\begin{aligned} i\,\dot{a} = &J\left(-\frac{3}{8}a'' - \frac{1}{4}a^\dagger a'' a - a' b'^\dagger b - \frac{1}{4}b''^\dagger ba - \frac{1}{2}b^\dagger ba''\right)\\ &+ J\left(\frac{1}{2}ab'^\dagger b' + \frac{1}{4}ab^\dagger b'' - \frac{1}{4}a''^\dagger a^2 - \frac{1}{2}a'^\dagger a' a\right)\\ &+\frac{1}{2}D\left(a^\dagger a - b^\dagger b + \frac{1}{2}\right)a - \frac{1}{2}hb \end{aligned}\quad (5)$$

$$\begin{aligned} i\dot{b} = &J\left(-\frac{3}{8}b'' - \frac{1}{4}b^\dagger b'' b - b' a'^\dagger a - \frac{1}{4}a''^\dagger ab - \frac{1}{2}a^\dagger ab''\right)\\ &+ J\left(\frac{1}{2}ba'^\dagger a' + \frac{1}{4}ba^\dagger a'' - \frac{1}{4}b''^\dagger b^2 - \frac{1}{2}b'^\dagger b' b\right)\\ &+\frac{1}{2}D\left(b^\dagger b - a^\dagger a + \frac{1}{2}\right)b - \frac{1}{2}ha, \end{aligned}\quad (6)$$

where for simplicity we assume an implicit dependence of the operators on the z and t variables, the dot defines $\partial/\partial t$, and the primes denote $\partial/\partial z$. In the classical limit of the spin variable, Schwinger coupled-bosons operators permit us to define the coherent states $|\alpha\beta\rangle$ [19] that are eigenstates of the bosonic operators a and b with eigenvalues α and β, respectively, so that it is possible to write the coupled differential equations for these eigenvalues by bracketing the equations for the bosonic operators into an arbitrary coherent state (Ferrer and Pozo [20]), obtaining the following two non-linear complex differential equations:

$$\begin{aligned} i\dot{\alpha} = &J\left(-\frac{3}{8}\alpha'' - \frac{1}{4}|\alpha|^2\alpha'' - \alpha'\beta'^*\beta\right)\\ &- J\left(\frac{1}{4}\alpha\beta''^*\beta - \frac{1}{2}|\beta|^2\alpha'' + \frac{1}{2}|\alpha'|^2\alpha\right)\\ &+ J\left(\frac{1}{2}\alpha|\beta'|^2 + \frac{1}{4}\alpha\beta^*\beta'' - \frac{1}{4}\alpha''^*\alpha^2\right)\\ &+\frac{1}{2}D\left(|\alpha|^2 - |\beta|^2 + \frac{1}{2}\right)\alpha - \frac{1}{2}h\beta, \end{aligned}\quad (7)$$

$$\begin{aligned} i\dot{\beta} = &J\left(-\frac{3}{8}\beta'' - \frac{1}{4}|\beta|^2\beta'' - \beta'\alpha'^*\alpha - \frac{1}{4}\beta\alpha''^*\alpha\right)\\ &+ J\left(\frac{1}{2}\beta|\alpha'|^2 + \frac{1}{4}\beta\alpha^*\alpha'' - \frac{1}{4}\beta''^*\beta^2\right)\\ &- J\frac{1}{2}\left(|\alpha|^2\beta'' + |\beta'|^2\beta\right)\\ &+\frac{1}{2}D\left(|\beta|^2 - |\alpha|^2 + \frac{1}{2}\right)\beta - \frac{1}{2}h\alpha, \end{aligned}\quad (8)$$

where $\alpha=\alpha(z,t)$ and $\beta=\beta(z,t)$. On the other hand, using the kinematics conditions associated with the Schwinger transformation:

$$\begin{aligned} |\alpha(z,t)|^2 - |\beta(z,t)|^2 &= 2\rho(z,t);\\ |\alpha(z,t)|^2 + |\beta(z,t)|^2 &= 2S, \end{aligned}\quad (9)$$

where $\rho(z,t)$ accounts for the out-of-plane spin deviations. Solving for α and β we get, to the first order in $\rho(z,t)$:

$$\begin{aligned} \alpha(z,t) &= \sqrt{S}(1+\rho(z,t)/2S)\exp[i\theta_\alpha(z,t)]\\ \beta(z,t) &= \sqrt{S}(1-\rho(z,t)/2S)\exp[i\theta_\beta(z,t)], \end{aligned}\quad (10)$$

where $\theta_\alpha(z,t)$ and $\theta_\beta(z,t)$ are real-angle variables.

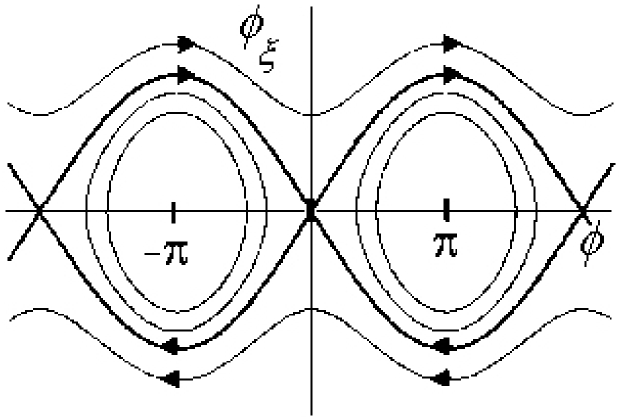

FIGURE 1. Phase plane diagram $\phi_\xi(\phi)$ for the reduced sine-Gordon equation, *i.e.* the nonlinear ODE obtained by assuming $\xi \equiv z - vt$.

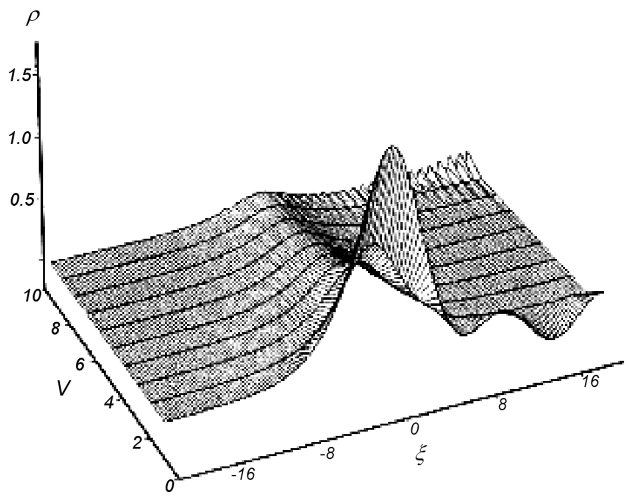

FIGURE 2. Out-of-plane spin deviations $\rho(\xi)$ versus ξ and velocity v. The values of the parameters are: $h = 0.1$ and $D = 0.2$.

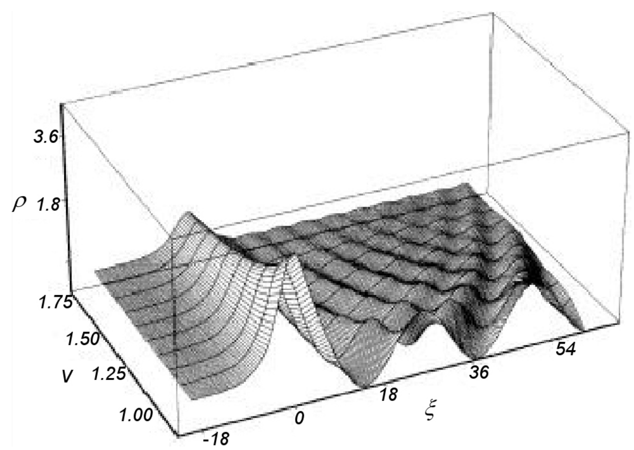

FIGURE 3. Out-of-plane spin deviations $\rho(\xi)$ versus ξ and velocity v.(low range of the velocity). The values of the parameters are $h = 0.1$ and $D = 0.2$

On the other hand, we note that the averages of the spin variables are given by:

$$\langle \alpha\beta | S^z(z,t) | \alpha\beta \rangle = 2\rho(z,t)$$
$$\langle \alpha\beta | S^x(z,t) | \alpha\beta \rangle = \Re(\alpha^\star \beta) = \cos(\theta_\beta - \theta_\alpha)$$
$$\langle \alpha\beta | S^y(z,t) | \alpha\beta \rangle = \Im(\alpha^\star \beta) = \sin(\theta_\beta - \theta_\alpha). \qquad (11)$$

3. Results

Proceeding as Ferrer and Pozo do [20] (from all these equations) after some algebraic manipulation, the real and imaginary parts can be separated; then, considering the case of permanent form solutions defining $\xi \equiv z - vt$ where v is a velocity, we obtain to the first order in ρ, θ_α and θ_β,

$$\phi_{\xi\xi} = \frac{h}{JSC_o} \sin\phi(\xi), \qquad (12)$$

where we have defined $\phi(\xi) = \theta_\beta(\xi) - \theta_\alpha(\xi)$, and $\phi_{\xi\xi} = d^2\phi/d\xi^2$.

So we find that the sine-Gordon equation reduces to the Ordinary Differential Equation (ODE). For $h < 0$, this would be the simple pendulum equation whose phase plane behaviour is well known. Let us look at the situation for $h > 0$. Setting $Y = \phi_\xi$ yields the two coupled first-order equations,

$$Y_\xi = \frac{JSC_0}{h} \sin\phi(\xi); \qquad \phi_\xi = Y.$$

Here, dividing to eliminate ξ, we find

$$\frac{dY}{d\phi} = \frac{JSC_o}{h} \frac{\sin\phi}{Y}$$

From the above equation, we obtain the phase plane diagram for the reduced sine-Gordon equation that is given by Fig. 1, where the heavy curves indicate the separatrixes connecting the saddle points. The arrows indicate the direction of increasing ξ. Kink solution (domain–wall solution) can be immediately spotted. The separatrix line for $\phi > 0$ and $Y = \phi_\xi$ connecting the origin and the saddle point corresponds to the domain-wall solution, which can be written as

$$\phi(\xi) = \arccos\left(1 - 2\mathrm{sech}^2\left(\sqrt{\frac{h}{C_o}}\xi\right)\right),$$

where $C_o = (3/8S + 1)$ [21].

We also find the following equation for the out-of-plane spin deviations:

$$\rho''(\xi) + G(\xi)\,\rho(\xi) \,=\, Q(\xi), \qquad (13)$$

where the derivatives are about ξ and we use the time as a parameter. We also define:

$$
\begin{aligned}
G(\xi) &= \frac{1}{C_o}[F(\xi) - H(\xi)] \\
H(\xi) &= \frac{\sqrt{3J}v\phi'^2(\xi)}{8S\,L(\xi)} \\
L(\xi) &= \sqrt{h\cos\phi(\xi) - \frac{1}{2}D - (\frac{3}{16} + S)J\phi'^2(\xi) + \frac{4v^2}{3J}} \\
Q(\xi) &= -\frac{\sqrt{3J}}{2C_o}\phi'(\xi)L(\xi).
\end{aligned}
\tag{14}
$$

Equation (13) shows that the out-of-plane spin deviations are governed by the linear inhomogeneous differential equation. To obtain the out-of-plane spin deviations $\rho(\xi)$ we perform a numerical integration (Pozo and Ferrer [22]), taking $J = 1$ and $S = 1$.

From Fig. 2 it is possible to observe that the critical behaviour corresponding to the regions where the out-of-plane spin deviation $\rho(\xi)$ loses instability, is due to the velocity of these nonlinear excitations (solitons). If we increase the velocity of the out-of-plane spin deviation $\rho(\xi)$, then it decays into high frequency-oscillations. For these values of the parameters we find that it is only possible to find the stability of $\rho(\xi)$ between $3 < v < 9$ (approximately). The same effect can be observed in Fig. 5 for the high range of the velocity. When the velocity becomes lower, the stable soliton corresponding to $\rho(\xi)$ is distorted by magnons and loses stability; this situation can be observed in Fig. 3.

In Fig. 4, the effects produced by the magnetic field h on the out-of-plane spin deviation $\rho(\xi)$are shown. Here, the values of the parameters are $v = 5$ and $D = 0.2$, where we can appreciate the fact that for small values of the magnetic field, $\rho(\xi)$ has a small amplitude and is distorted by magnons. Nevertheless, when the magnetic field increases, we find a region of stability between $h = 0.64$ and $h = 1.28$, where $\rho(\xi)$ increases his amplitude, but after the region $\xi = 0$, it loses stability and is accompanied by oscillations of the magnon type.

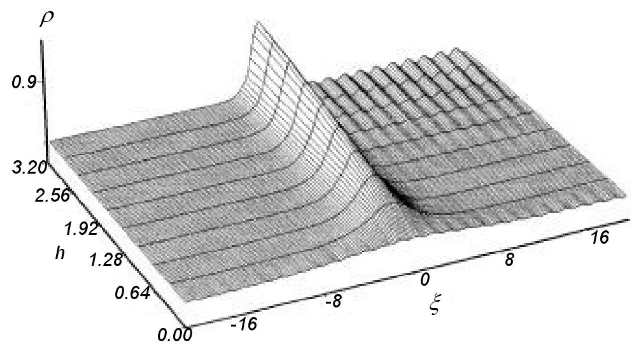

FIGURE 4. Out-of-plane spin deviations $\rho(\xi)$ versus ξ and the magnetic field h. The values of the parameters are $v = 6.0$ and $D = 0.2$

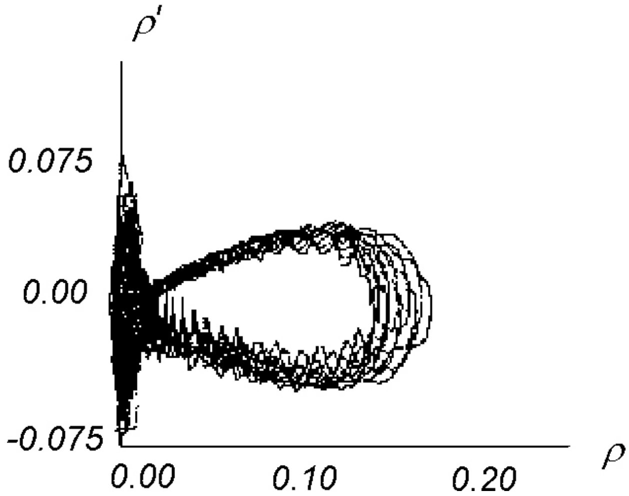

FIGURE 5. Phase plane of out-of-easy-plane deviations $\rho'(\xi)$ (high range of velocity). The values of the parameters are $h = 0.1$, $d \equiv D/J = 0.2$.

In Fig. 5, we show the phase plane of out-of-easy-plane deviations $\rho'(\xi)$, for the high range of velocity, where it is possible to appreciate the critical behaviour of the instability produced by the velocity, and we observe the form in which $\rho(\xi)$ decays into high-frequency-oscillations [23,26].

4. Summary and concluding remarks

We have considered in this paper a ferromagnetic Heisenberg chain with easy-plane anisotropy in the presence of an external magnetic field h parallel to that plane, in the classical continuum limit. Our interest was focussed on investigating and describing the existence and the behaviour of out-of-plane spin deviation $\rho(\xi)$ in this system by using the coupled-bosons operators together with the Schwinger transformation for the spin operator. The results obtained may be summarized as follows:

We find that the out-of-plane spin deviation $\rho(\xi)$ is governed by the inhomogeneous linear differential equation (13). A similar equation was already obtained by Magyari and Thomas [4] for the low velocity limit.

The formalism considered in this paper permits us to obtain (as a particular case) the sine-Gordon static case which is given by Eq. (12), from phase plane diagram $\phi_\xi(\phi)$, Fig. 1. The separatrix line connecting the origin and the saddle point corresponds to the domain–wall solution for the sine-Gordon equation. The reason why the phase plane is analysed is that the relevant basic assumption that permanent form solutions defining $\xi \equiv z - vt$ reduce the nonlinear Partial Differential Equation (PDE) to a nonlinear Ordinary Differential Equation (ODE).

We observe that the critical behaviour of the instability of the out-of-plane spin deviation is due to low and high ranges of velocity of these nonlinear excitations (solitons), which can be appreciated in Figs. 2 and 3. These regions of instability of the soliton can be the ones that produce the diverging out-of-plane spin deviation [6].

We find an opposite competence effect produced by the velocity v and the magnetic field h on the amplitude of out-of-plane spin deviation $\rho(\xi = 0)$, which can be appreciated by comparing Figs. 2 or 3 with 4. The amplitude values of $\rho(\xi = 0)$ decrease when the velocity v increases, and the amplitude of $\rho(\xi = 0)$ increases when the magnetic field h increases.

Finally, as shown, the existence and the behaviour of out-of-plane spin deviation $\rho(\xi)$ play an important role in the dynamic of this system, and permit an agreement with the experimental interpretation [2].

1. J.P. Boucher *et al.*, *Solid State Comm.* **33** (1980) 171.
2. H. Seit and H. Berner, *Z. Phys. B* **66** (1987) 485.
3. H.J. Mikeska, *J. Phys. C* **11** (1978) L29.
4. E. Magyari and H. Thomas, *Phys. Rev. B* **25** (1982) 531.
5. P. Kumar, *Physica D* 5 (1982) 359.
6. H.J. Mikeska and Osano, *Z. Phys. B* **52** (1983) 111.
7. B.A. Ivanov and H.J. Mikeska, *Phys. Rev. B* **70** (2004) 174409.
8. J.M. Weisselinowa and V.P. Ivanova, *Phys. Rev B* **54** (1996) 9846.
9. K.K. Muminov and V.K. Fedynanin, *Physica Scripta* **62** (2000) 23.
10. F.E. Galaly *et al.*, *PRL* **96** (2006) 147202.
11. G.N. Kakazei *et al*., *J. of Appl. Phys 93* (2003).
12. J.A.C. Bland *et al*., *J. Phys. D: Appl. Phys* **35** (2002).
13. J.M. Wesselinowa and V.P. Ivanova, *Phys. Rev. B Cond. Matt.* **54** (1996) 14.
14. S.L. Menezes, A.S. Pires, and M.E. Gouvea, *Phys. Rev. B Cond. Matt.* **45** (1992) 18.
15. F.C. Alcaraz, A. Saguia, and M.S. Sarandy, *Phys. Rev. A* **70** (2004) 032333.
16. G.N. Kakasei *et al., J. of Appl. Phys.* **93** (2003).
17. A.T. Boothroyd *et al*., *Phys. Rev. Lett.* **92** (2004) 19.
18. F. El Galaby *et al*., *Phys. Rev. Lett.* **96** (2006).
19. R. Ferrer and J. Pozo, *Physica B* **153** (1988) 225.
20. R. Ferrer and J. Pozo, *J. Phys.: Cond. Matter* **3** (1991) 3527.
21. E. Magyari and H. Thomas, *Phys. Review B* **29** (1984) 6358.
22. J. Pozo and R. Ferrer, *New Trends in Magnetism, Magnetic Materials, and Their. Applications* (Plenum Press, New York and London (1994) p. 69.
23. R.H.G. Helleman, *Universality in Chaos*, P. Cvitanovic (ed). Adam Hilger (Bristol 1984).
24. E. Infeld and G. Rowlands, *Nonlinear Waves, Solitons and Chaos* (Cambridge University Press 2000).
25. S.H. Strogatz, *Nonlinear Dynamics and Chaos* (Addison Wesley 1994).
26. R.H. Enns and G.C. McGuire, *Nonlinear Physics*, Birkhäuser (Boston, Basel, Berlin 2001).