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The bootstrap method: an alternative for estimating confidence intervals of resources surveyed by hydroacoustic techniques[1]

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In order to obtain approximate confidence intervals of resources surveyed by hydroacoustic methods, the bootstrap method may be used as a valid alternative to the conventional method with a standard normal interval. The two methods are compared for a ratio estimator employing information from 11 seasonal surveys designed to estimate sardine (*Sardinops sagax*) biomass off northern Chile, during the period 1981–1985 in an area from the coast to 100 nautical miles offshore between Arica (18°20′S) and Antofagasta (23°00′S), Chile.

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Introduction

The ratio estimator ($\hat{R} = X/Y$) has been regularly used to estimate fish-population biomass (biomass = area × R) by means of hydroacoustic techniques. Shotton and Dowd (1975) suggested this estimator in a cluster sample design (Hansen *et al.*, 1953) to provide lower estimated variances than alternative designs.

Williamson (1982), by means of simulation, compared cluster sampling with simple random sampling. He determined that the most reliable confidence intervals can be obtained using cluster sampling estimation techniques. The construction of confidence intervals of one estimator represents a more ambitious measure of statistical accuracy than the standard error. Until now, the usual procedure for constructing an interval for R, has been to use the approximate form of a standard interval R: $\hat{R} \pm SD(\hat{R}) \times Z_\alpha$, where Z_α is the 100_α percentile point of a standard normal distribution.

This method of interval computation assumes asymptotic normality, suggesting that the normal approximation applies reasonably well if the sample is large (Cochran, 1971), in which case the distribution of $\hat{R}$ becomes symmetric as does its confidence interval. However, in acoustics the estimates are obtained with relatively small samples for resources that can have highly aggregated distributions. It is therefore necessary to consider the validity of applying traditional statistical analyses if some of the assumptions are not acceptable.

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We compared confidence intervals estimated for sardine (*Sardinops sagax*) biomass off the shores of northern Chile, using two methods: the conventional method, based on a standard interval subject to the Kish analytic expression (Williamson, 1982) for the ratio estimator $\hat{R}$; and the bootstrap BC method (Efron, 1982) for obtaining confidence intervals.

The data were collected during a total of 11 seasonal surveys conducted between 1981 and 1985. The survey area covered the region between Arica and Antofagasta, Chile, from the immediate offshore area to 100 nautical miles from the coast.

Methods

The confidence intervals estimated for the R ratio, using conventional sampling-theory methods in a random sampling design of unequal-size clusters and assuming asymptotic normal distribution, were compared with the alternative of bootstrap confidence intervals stemming from the Bias-Corrected Percentile Method, or BC method (Efron, 1982; introduced by Efron, 1979). The bootstrap is a general methodology based on a computational method. Due to the fact that it is not necessary to assume normality, Buckland (1984) describe the method as "partially parametric" or "fully nonparametric", depending on whether the density estimator is or is not parametric. This method allows an investigator to address problems that are too complex for traditional statistical analysis, such as determination of the accuracy of a particular estimator. It can be justified because it permits the elimination of two limiting factors: the assumption that the data conform to a bell-shaped curve, and the need to focus on statistical measures whose theoretical properties can be analysed mathematically (Diaconis and Efron, 1983; Haslett and Wear, 1985).

Efron (1982) and Efron and Tibshirani (1986) define at least three different types of confidence intervals, one of which is used in this study, the Bias-Corrected Percentile Method (BC method). Efron (1982) shows that when the assumption of symmetry fails, the percentile method can be corrected for bias by means of a distribution transformation.

The BC method employs the percentiles of the accumulated distribution function, $\hat{G}(S)$, defined by:

$$\hat{G}(S) = \text{Prob}_*(\hat{R}^* \leq S)$$

where Prob_* indicates probability computed according to the bootstrap distribution of $\hat{R}^*$, and $\hat{R}^*(b)$ is a vector of ratio estimators, evaluated in each bootstrap sample $(b = 1, 2, \ldots, B)$. A bootstrap sample turns out to be the same as a random sample of size n, drawn with replacement from the actual sample $[(X_1, Y_1), (X_2, Y_2), \ldots, (X_i, Y_i), \ldots, (X_n, Y_n)]$, where X is the density of the (i)th transect and Y_i is the number of observations in the (i)th transect.

A bias correction should be used when the $\text{Prob}_*(\hat{R}^* \leq \text{Prob}_*) \neq 0.5$, being $\hat{R}$, the estimation for actual sample data.

If we suppose that M of the B bootstrap replications leads to estimates of R that are smaller than $\hat{R}$, then we define:

$$Z_0 = \varnothing^{-1}[\hat{G}(\hat{R})] \quad \text{with} \quad \hat{G}(\hat{R}) = M/B$$

where $\varnothing^{-1}$ is the inverse function of the standard normal distribution. Then the BC method consists of taking:

$$\{\hat{G}^{-1}[\varnothing(2Z_0 - Z_\alpha)];\ \hat{G}^{-1}[\varnothing(2Z_0 + Z_\alpha)]\}$$

as an approximate $1-2\,\alpha$ central confidence interval for R, where Z_α is the upper α point for a standard normal $\varnothing(Z_\alpha) = 1-\alpha$.

For more details see Efron (1982), Buckland (1984), and Efron and Tibshirani (1986).

Results and discussion

The series of 11 surveys analysed in this paper have established that there is a clear seasonality in sardine availability (Fig. 1) which reaches a maximum in winter when catches are higher. We applied a Monte Carlo simulation procedure to data from each survey, with a total of 7500 bootstrap replications. The number of transects with which the empirical distribution $\hat{G}(S)$ was obtained ranged between 14 and 27, sample sizes that may be considered moderately small. Of the $\hat{G}(S)$ distributions generated (Fig. 2), nine present positive asymmetry, one presents negative asymmetry, and one is symmetric.

When the confidence intervals computed by the two methods are compared for each survey (Table 1), there are some differences, but they are small and not of significance in the context of using the data for management purposes. The BC method tends to produce an interval shifted to the right of the symmetric interval because the majority of the $\hat{G}(S)$ empirical distributions are positively asymmetrical. Interval widths are similar,

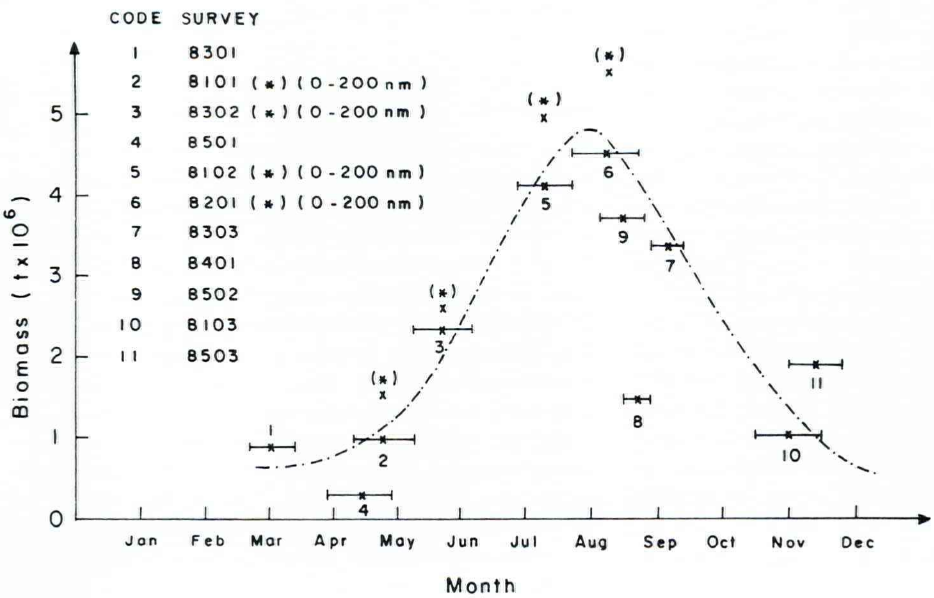

Figure 1. Acoustic biomass (t) of sardine (*Sardinops sagax*) between Arica and Antofagasta, from the coast to 100 miles offshore. In the column for surveys the first two digits show the year and the last two the order of occurrence. The horizontal bars represent the periods during which the surveys were carried out.

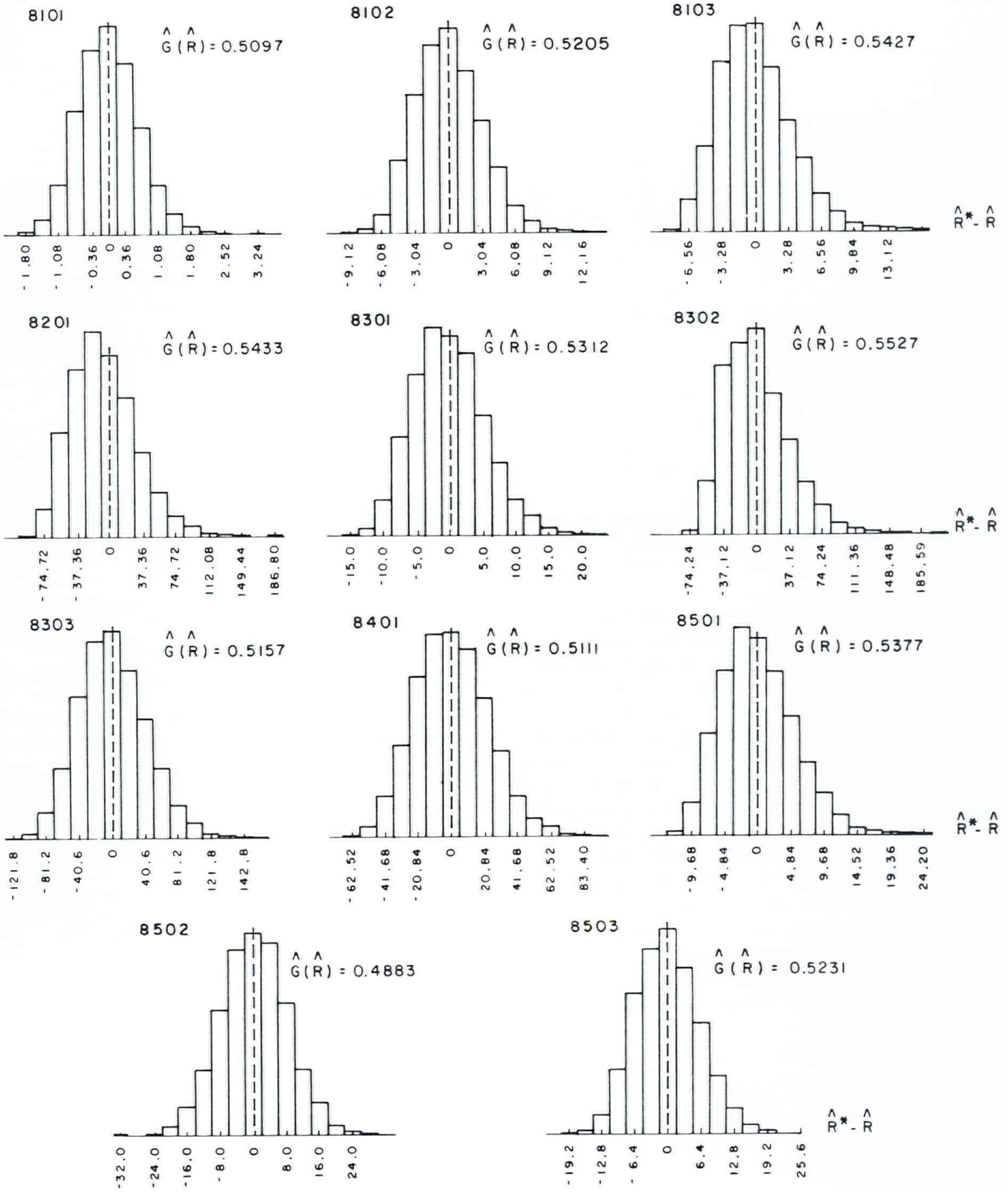

Figure 2. Histogram of B = 7500 bootstrap replications of $\hat{R}^* - \hat{R}$ by survey. The histogram for $\hat{R}^*$ is centred on $\hat{R}$.

Table 1. Estimated approximate 90 % confidence intervals for R by survey.

Survey[a]	Sample size	Confidence intervals	
		Standard method	Bootstrap method (BC)
8101	27	[2.73; 4.87]	[2.82; 4.94]
8102	20	[10.13; 20.25]	[10.88; 20.68]
8103	21	[4.99; 16.67]	[6.12; 17.79]
8201	21	[53.06; 176.16]	[64.68; 187.65]
8301	22	[12.94; 31.00]	[14.35; 32.40]
8302	16	[61.33; 179.67]	[80.98; 202.02]
8303	19	[136.17; 275.66]	[148.15; 284.76]
8401	20	[36.48; 112.68]	[40.34; 115.15]
8501	14	[4.87; 22.97]	[6.76; 24.45]
8502	14	[45.21; 71.19]	[45.27; 71.52]
8503	14	[16.80; 37.74]	[18.30; 38.35]

[a]First two digits: year; last two digits: order of occurrence during the year.

but BC-method intervals are generally slightly narrower.

The main advantage of the BC percentile method is that it is not necessary to make any assumptions regarding the data, since the model may be considered as derived from the data itself rather than imposed on the data. Further, it is not necessary to calculate the variance structure of the estimator, a procedure which is often analytically complex.

The interval of the BC method is obtained from the $\hat{G}(S)$ empirical distribution, and it is impossible to compute limits that fall outside the range of the actual sample. However, the standard interval method could give an inconsistent biomass confidence interval under some circumstances. This possibility is greatest when the resource under survey is clustered and the sample size is small. If it is necessary to increase the number of surveys without increasing the budget, then the number of transects must be reduced. In this case, if we want to apply the conventional method for calculating a standard confidence interval, the distribution of data in the original population should be considered. Barrett and Goldsmith (1976) argued that when the distribution of data is approximately normal, sample sizes can be small, but if the distribution is highly skewed, sample sizes must be large. The bootstrap confidence-interval method coincides asymptotically with the standard confidence interval since it is invariant under transformation. In consequence, the bootstrap method for constructing confidence intervals provides a good alternative for estimation, especially when analytical intervals are not available or are unreliable and the sample sizes are small. The lower or null theoretical analysis requires substantial computational effort (Efron and Tibshirani, 1986).

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