Warped DGP model in warm intermediate inflation with a general dissipative coefficient in light of BICEP2 and Planck results

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A warm inflationary universe scenario on a warped Dvali-Gabadadze-Porrati brane during intermediate inflation is studied. We consider a general form for the dissipative coefficient $\Gamma(T,\phi) \propto T^m/\phi^{m-1}$, and also study this model in the weak and strong dissipative regimes. We analyze the evolution of the Universe in the slow-roll approximation and find the exact solutions to the equations of motion. In both regimes, we utilize recent data from the BICEP2 experiment and also from the Planck satellite to constrain the parameters in our model in accordance with the theory of cosmological perturbations.

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I. INTRODUCTION

It is well known that the scenario of warm inflation is different than that of traditional cold inflation, notably in that warm inflation avoids a reheating period [1]. During the evolution of warm inflation, dissipative effects are important, and radiation effects take place at the same time as the expansion of the Universe. The dissipating effect occurs due to a friction term which accounts for the processes of the scalar field dissipating into a thermal bath. In further relation to these dissipative effects, the dissipative coefficient Γ is a fundamental quantity. This parameter Γ was studied in a supersymmetric model [2] for a low-temperature scenario. For a scalar field with multiplets of heavy and light fields that give different expressions for the dissipation coefficient, see Refs. [2–7]. A general form for the dissipative coefficient $\Gamma(T,\phi)$, can be written as [5,6]

$$\Gamma(T,\phi) = C_\phi \frac{T^m}{\phi^{m-1}}, \tag{1}$$

where the constant C_ϕ is associated with the dissipative microscopic dynamics and the constant m is an integer. Various values of m have been considered in the literature (see Refs. [5,6]). Specifically, for the value of $m=3$, i.e., $\Gamma \propto T^3\phi^{-2}$, the parameter C_ϕ corresponds to $C_\phi = 0.02h^2\mathcal{N}_Y$, where a generic supersymmetric model with chiral superfields Φ, X and Y_i, $i=1,\ldots\mathcal{N}_Y$ is considered [7,8]. For the special value $m=1$, Γ is associated with the high-temperature supersymmetry (SUSY) case. For the special case $m=0$, the dissipation coefficient represents an exponentially decaying propagator in the high-temperature SUSY model. For the value $m=-1$, i.e., $\Gamma \propto \phi^2/T$, agrees with the non-SUSY case [3,9]. Additionally, thermal fluctuations during the inflationary scenario may play a fundamental role in producing the initial fluctuations essential for large-scale structure (LSS) formation [10,11]. During the warm inflationary scenario the density perturbations arise from the thermal fluctuations of the scalar field and dominate over the quantum origin of the initial density perturbations. In this form, an essential condition for warm inflation scenario is the existence of a radiation component with temperature $T > H$, during the expansion of the Universe, since the thermal and quantum fluctuations are proportional to T and H, respectively [1,10,11]. Also relevant, as the Universe heats up and becomes radiation dominated, then warm inflation ends. Here, the Universe stops inflating and smoothly enters in a radiation Big-Bang phase [1]. For a comprehensive review of warm inflation, see Ref. [12].

On the other hand, from high-dimensional gravity theory, Dvali-Gabadadze-Porrati (DGP) considered a braneworld model [13] where the Universe is a four-dimensional brane embedded in a five-dimensional Minkowski space-time. In this perspective, the induced gravity brane-world in the DGP model was put forward as an alternative to the Randall-Sundrum (RS) one-brane model [14]. The gravitational behaviors in the DGP model are divided between the five-dimensional curvature scalar in the bulk and the four-dimensional curvature scalar on the brane. According to the embedding of the brane in the bulk in the DGP brane, there are two branches of background solutions; i.e., there are two forms to embed space-time (the four-dimensional brane into the five-dimensional space). The inflationary universe model in the context of warped DGP has been analyzed in Refs. [15–19]. In particular, the

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warm inflation model in a DGP scenario was analyzed in Ref. [20], where the authors studied a standard scalar field coupled to radiation for an effective exponential potential. Also, the cosmological dynamics of a quintom field on the warped DGP brane was analyzed in Ref. [21], and observational constraints on the normal branch of a warped DGP cosmology were found in Ref. [22]. For a review of the DGP model, see Ref. [23].

On the other hand, in the context of the exact solutions, can be obtained for instance in the intermediate inflation model [24], where the scale factor is slower than de Sitter expansion, but quicker than power law (hence the name "intermediate"). During this scenario, the scale factor grows as

$$a(t) = \exp[At^f], \tag{2}$$

where A and f are two constants, and in which $A > 0$ and the constant f varies between $0 < f < 1$ [24]. It is well known that the exact solutions in inflationary scenarios can be obtained from an exponential potential or "power-law" inflation, in which $a(t) \sim t^p$, where $p > 1$ [25]. Similarly, an exact solution can be found from a constant potential, de Sitter inflation [26]. In the context of the intermediate model, the scale factor $a(t)$ was earlier elaborated as an exact result of the equation of motion, but the dynamics of the model may be best described from slow-roll approximation. In the slow-roll approximation, it is feasible to obtain a spectral index $n_s \sim 1$ (and in particular for the specific value of $f = 2/3$, the spectral index results $n_s = 1$, or the Harrizon-Zel'dovich spectrum). Likewise, an important observational magnitude found in the intermediate model is the tensor-to-scalar ratio r, which becomes $r \neq 0$ [27].

Thus the aim of the paper is to study an intermediate scale factor during warm inflation scenario in the framework of a warped DGP model, and how a generalized form of dissipative coefficient $\Gamma(T,\phi) \propto T^m/\phi^{m-1}$ influences our model. We will consider a warm intermediate inflation on a warped DGP model for two regimes; the weak and the strong dissipative scenarios, respectively. Also, we will investigate the cosmological perturbations, which are expressed in terms of different parameters appearing in our model. These parameters are constrained by the BICEP2 experiment data [28] and the Planck satellite [29]. The BICEP2 results imply a large amplitude of primordial gravitational waves and hence has important theoretical significance on inflationary universe models. The observational data on the tensor-to-scalar ratio r, has been obtained at more than 5σ confidence level (C.L.) with a rigorous constraint, where $r = 0.20^{+0.07}_{-0.05}$ at 68% C.L., also $r = 0.16^{+0.06}_{-0.05}$ with foreground subtracted. However, the ratio r has become less clear when grave criticisms to the BICEP2 analysis appeared in the literature. Recently, the Planck Collaboration has issued the data relating the polarized dust emission [30]. Here, an analysis of the polarized thermal emissions from diffuse Galactic dust in the range of 353 to 150 GHz suggests that BICEP2 gravitational wave result could be due to the dust contamination, and a detailed study of Planck and BICEP2 data would be required for a definitive answer.

The outline of the paper is as follows. The next section presents a short review of the Friedmann equation on the warped DGP inflation model. In Sec. III we present the warm inflationary phase on the warped DGP model, study the weak and strong dissipative regimes, and discuss the inflationary epoch and the cosmological perturbations in both regimes. Finally, Sec. IV summarizes our findings. We choose units so that $c = \hbar = 1$.

II. THE FRIEDMANN EQUATION ON THE WARPED DGP BRANE

The Friedmann equation on the warped DGP model can be written from the Friedmann-Robertson-Walker (FRW) metric as

$$H^2 = \frac{1}{3\mu^2}[\rho + \rho_0(1 + \epsilon\mathcal{A}(\rho, a))], \tag{3}$$

where $H = \dot{a}/a$ corresponds to the Hubble parameter and ρ is the total energy density. Here, the dots mean derivatives with respect to time. The constant μ denotes the strength of the induced gravity term on the brane (the special case when $\mu = 0$ yields the RS model [14]). The parameter ϵ corresponds to $+1$ or -1, which are the two branches of the warped DGP brane. For the value $\epsilon = -1$, we will consider the brane tension as positive, and for the value $\epsilon = +1$, negative. Here, the function $\mathcal{A}(\rho, a)$ is defined as

$$\mathcal{A} = \left[\mathcal{A}_0^2 + \frac{2\eta}{\rho_0}\left(\rho - \mu^2\frac{\mathcal{E}_0}{a^4}\right)\right]^{\frac{1}{2}}, \tag{4}$$

where the constants $\mathcal{A}_0$, ρ_0, and η are given by $\mathcal{A}_0 = \sqrt{1 - 2\eta\frac{\mu^2\Lambda}{\rho_0}}$, $\rho_0 = m_\lambda^4 + 6\frac{m_5^6}{\mu^2}$, $\eta = \frac{6m_5^6}{\rho_0\mu^2}$ $(0 < \eta \leq 1)$, and the constant Λ becomes $\Lambda = \frac{1}{2}({}^{(5)}\Lambda + \frac{1}{6}\kappa_5^4\lambda^2)$. Here, κ_5 is the five-dimensional Newton constant, ${}^{(5)}\Lambda$ corresponds to the five-dimensional cosmological constant in the bulk, $\mathcal{E}_0$ is a constant related to Weyl radiation, and the brane tension is specified by λ. Here, there are three mass scales, μ, $m_\lambda = \lambda^{1/4}$, and $m_5 = \kappa_5^{-2/3}$. Since we are only concerned with inflationary dynamics in our model, we will ignore the dark radiation term. Also, we shall restrict ourselves to the RS critical case, where $\Lambda = 0$. In this form, Eq. (3) yields

$$H^2 = \frac{1}{3\mu^2}\left[\rho + \rho_0 + \epsilon\rho_0\left(1 + \frac{2\eta\rho}{\rho_0}\right)^{1/2}\right]. \tag{5}$$

Note that in the ultrahigh energy limit in which $\rho \gg \rho_0 \gg m_\lambda^4$, the Friedmann equation given by Eq. (5) becomes $H^2 \propto (\rho + \epsilon\sqrt{2\rho\rho_0})$ and corresponds to four-dimensional gravity on the brane. Also, in the intermediate energy region in which $\rho \ll \rho_0$ but $\rho \gg m_\lambda^4$, for the branch with $\epsilon = -1$, Eq. (3) becomes $H^2 \propto (\rho + \frac{\rho^2}{2m_\lambda^4} - \frac{\mu^2 m_\lambda^4}{6m_5^6}\rho - \frac{\mu^2}{4m_5^6}\rho^2)$, and at the low energy limit, where $\rho \ll m_\lambda^4 \ll \rho_0$ then Eq. (3) becomes $H^2 \propto [\rho + \mathcal{O}(\rho/\rho_0)^2]$, where μ_p is the effective four-dimensional Planck mass and $\mu_p^2 = \mu^2/(1-\eta)$.

III. WARM INFLATION: BASIC EQUATIONS

In the following, we consider the Universe filled with a self-interacting scalar field of energy density ρ_ϕ together with a radiation field given by ρ_γ. In this form, the total energy density ρ can be written as $\rho = \rho_\phi + \rho_\gamma$. In the following, we will assume that the energy density associated with the scalar field is $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$ and the pressure is $P_\phi = \dot{\phi}^2/2 - V(\phi)$, where the quantity $V(\phi)$ represents the effective potential.

Also, we will assume that the total energy density ρ is confined to the brane in the bulk satisfying the continuity equation given by $\dot{\rho} + 3H(\rho + P) = 0$. In this form, the dynamical equations for ρ_ϕ and ρ_γ are described by [1]

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = -\Gamma\dot{\phi}^2, \qquad (6)$$

and

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\dot{\phi}^2, \qquad (7)$$

where the dissipation coefficient $\Gamma > 0$ [see Eq. (1)], and produces decay of the scalar field into radiation. Recall that the parameter Γ can be assumed to be a function of the temperature of the thermal bath $\Gamma(T)$, or a function of the scalar field $\Gamma(\phi)$, or a function of $\Gamma(T,\phi)$ or simply a constant [1].

In the context of warm inflation, the energy density related to the scalar field predominates over the energy density of the radiation field, i.e., $\rho_\phi \gg \rho_\gamma$ [1,10,31–33] and then $\rho \sim \rho_\phi$. In this approximation, Eq. (5) can be written as

$$H^2 \approx \frac{1}{3\mu^2}\left[\rho_\phi + \rho_0 + \epsilon\rho_0\left(1 + \frac{2\eta\rho_\phi}{\rho_0}\right)^{1/2}\right],$$

or equivalently as

$$H^2 \approx \frac{1}{3\mu^2}\left[\frac{\dot{\phi}}{2} + V(\phi) + \rho_0 + \epsilon\rho_0\left(1 + \frac{\eta[\dot{\phi} + 2V(\phi)]}{\rho_0}\right)^{1/2}\right]. \qquad (8)$$

From Eqs. (6) and (8), we get

$$\dot{\phi}^2 = 2\mu^2\frac{(-\dot{H})}{(1+R)}[1 - \epsilon(\alpha + \beta H^2)^{-1/2}], \qquad (9)$$

where $R = \frac{\Gamma}{3H}$ denotes the rate between Γ and the Hubble parameter. Note that for the case of the weak or strong dissipation regime, we make $R < 1$ or $R > 1$, respectively. Following Ref. [19], the constants α and β are defined by

$$\alpha = 1 + \frac{\mathcal{A}_0^2}{\eta^2} - \frac{2}{\eta}, \quad \text{and} \quad \beta = \frac{6\mu^2}{\eta\rho_0}.$$

On the other hand, during the inflationary scenario, we assume that radiation production is quasistable, i.e., $\dot{\rho}_\gamma \ll 4H\rho_\gamma$ and $\dot{\rho}_\gamma \ll \Gamma\dot{\phi}^2$ (see Refs. [1,10,31–33]). In this form, by using Eqs. (7) and (9), the density ρ_γ, becomes

$$\rho_\gamma = \frac{\Gamma\dot{\phi}^2}{4H} = \frac{\mu^2\Gamma(-\dot{H})}{2H(1+R)}[1 - \epsilon(\alpha + \beta H^2)^{-1/2}]. \qquad (10)$$

Also, the energy density of the radiation field could be written as $\rho_\gamma = C_\gamma T^4$, where the constant $C_\gamma = \pi^2 g_*/30$. Here, g_* represents the number of relativistic degrees of freedom. Combining Eq. (10) with $\rho_\gamma \propto T^4$, we get

$$T = \left[\frac{\mu^2\Gamma(-\dot{H})}{2C_\gamma H(1+R)}\right]^{1/4}[1 - \epsilon(\alpha + \beta H^2)^{-1/2}]^{1/4}. \qquad (11)$$

In particular for the weak dissipative regime in which $R < 1$, from Eqs. (1) and (11), the temperature of the thermal bath T becomes

$$T = \left(\left[\frac{C_\phi\mu^2\phi^{1-m}(-\dot{H})}{2C_\gamma H}\right][1 - \epsilon(\alpha + \beta H^2)^{-1/2}]\right)^{\frac{1}{4-m}}, \qquad (12)$$

and for the strong dissipative regime ($R > 1$), the temperature is given by

$$T = \left[\frac{3\mu^2(-\dot{H})}{2C_\gamma H}\right]^{1/4}[1 - \epsilon(\alpha + \beta H^2)^{-1/2}]^{1/4}. \qquad (13)$$

On the other hand, the scalar potential V can be found combining Eqs. (8) and (9),

$$V = \frac{\eta\rho_0}{2}(\alpha + \beta H^2)[1 - \epsilon(\alpha + \beta H^2)^{-1/2}]^2 - \frac{\mathcal{A}_0^2\rho_0}{2\eta} + \frac{\mu^2\dot{H}}{(1+R)}\left(1 + \frac{3}{2}R\right)[1 - \epsilon(\alpha + \beta H^2)^{-1/2}], \qquad (14)$$

and this effective potential could also be obtained explicitly in terms of the scalar field, i.e., $V = V(\phi)$.

Also, the dissipation coefficient, by using Eqs. (1) and (11), can be rewritten as

$$\Gamma^{\frac{4-m}{4}}(1+R)^{\frac{m}{4}} = C_\phi\left[\frac{\mu^2}{2C_\gamma}\right]^{\frac{m}{4}}\phi^{1-m}\left[\frac{-\dot{H}}{H}\right]^{\frac{m}{4}} \times [1-\epsilon(\alpha+\beta H^2)^{-1/2}]^{\frac{m}{4}}. \quad (15)$$

Note that Eq. (15) fixes the dissipation coefficient in the weak (or strong) dissipative regime as a function of the scalar field ϕ (or the cosmological time t).

In the following, we will consider our model for a dissipative coefficient $\Gamma = \Gamma(T,\phi)$ given by Eq. (1), and we will restrict ourselves to the weak and strong dissipation regimes.

A. The weak dissipative regime

Assuming that our model evolves in agreement with the weak dissipative regime, where $\Gamma < 3H$, and using Eqs. (2) and (9), we obtain

$$\phi(t) - \phi_0 = \frac{F[t]}{K}, \quad (16)$$

where $\phi(t=0) = \phi_0$ is an integration constant, that can be assumed $\phi_0 = 0$. The constant K is defined by $K \equiv a_f(\frac{1-f}{2\mu^2 Af})^{1/2}(\beta A^2 f^2)^{-a_f/2}$ and the function $F[t]$ corresponds to the incomplete Lauricella function [34], defined as

$$F[t] \equiv \left(\alpha + \frac{\beta}{t^{2(1-f)}}\right)^{\frac{-a_f}{2}} F_D^{(3)}\left[a_f; 1+\frac{a_f}{2}, 1+\frac{a_f}{2}, \frac{-1}{2}, a_f+1, \sqrt{\alpha}, -\sqrt{\alpha}, \epsilon\left(\alpha+\frac{\beta}{t^{2(1-f)}}\right)^{\frac{-1}{2}}\right],$$

where the constant a_f is given by $a_f = \frac{f}{2(1-f)}$. The Hubble parameter as a function of the inflaton field, ϕ, from Eq. (16) becomes $H(\phi) = \frac{Af}{(F^{-1}[K\phi])^{1-f}}$, where F^{-1} represents the inverse function of the incomplete Lauricella function [34].

From Eq. (14), the scalar potential as function of the scalar field becomes

$$V(\phi) \simeq \frac{\eta\rho_0}{2}\left(\alpha + \frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}}\right) \times \left[1-\epsilon\left(\alpha+\frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}}\right)^{-1/2}\right]^2 - \frac{\mathcal{A}_0^2\rho_0}{2\eta}. \quad (17)$$

Here, we considered that only the first term of Eq. (14) predominates at large values of scalar field ϕ. Also, we observed that we would have found the same scalar potential given by Eq. (17) using the slow-roll approximations i.e., $\dot{\phi}^2 \ll V(\phi)$ and $\ddot{\phi} \ll 3H\dot{\phi}$.

That said, introducing the dimensionless slow-roll parameter ε, defined as $\varepsilon = -\frac{\dot{H}}{H^2} = \frac{1-f}{Af(F^{-1}[K\phi])^f}$, and with the requirement for inflation to take place, $\varepsilon < 1$ (or equivalently $\ddot{a} > 0$), we get $\phi > \frac{1}{K}F[(\frac{1-f}{Af})^{1/f}]$. Also, assuming that the inflationary scenario starts at the earliest possible scenario, in which $\varepsilon = 1$ (see Ref. [27]), the field ϕ_1 can be written as $\phi_1 = \frac{1}{K}F[(\frac{1-f}{Af})^{1/f}]$.

Introducing the number of e-folds N among two values of cosmological times t_1 and t_2, or equivalently among two different values ϕ_1 and ϕ_2, we get

$$N = \int_{t_1}^{t_2} H dt = A[(F^{-1}[K\phi_2])^f - (F^{-1}[K\phi_1])^f]. \quad (18)$$

Here we have considered Eqs. (2) and (16).

In the following, we will study the scalar and tensor perturbations in the weak dissipative regime for the warm warped DGP brane. Following Refs. [1,35], the density perturbation is given by $\mathcal{P}_\mathcal{R}{}^{1/2} = \frac{H}{\dot{\phi}}\delta\phi$. Here, we consider the gauge invariant quantity $\zeta = H + \delta\rho/\dot{\rho}$, which is defined on slices of uniform density and contracts to the curvature perturbation. A characteristic of this gauge invariant is that it is closely constant on super-horizon scales and does not depend on gravitational dynamics [36] (see also, Ref. [37]). In this case, the spectrum associated with the curvature spectrum could be written as $\mathcal{P}_\mathcal{R}{}^{1/2} \simeq \sqrt{\langle\zeta^2\rangle} \simeq \frac{H}{\dot{\phi}}\delta\phi$, which persists unchanged in the warped DGP model [38].

To continue in the scenario warm inflation, a thermalized radiation component exists and the fluctuations $\delta\phi$ are predominantly thermal instead of quantum. For the weak dissipative regime, the amplitude of the scalar field fluctuation is given by $\delta\phi^2 \simeq HT$ [10]. In this form, by using Eqs. (9) and (11), the power spectrum $\mathcal{P}_\mathcal{R}$, results in

$$\mathcal{P}_\mathcal{R} = \frac{\sqrt{3\pi}}{4\mu^2}\left(\frac{\mu^2 C_\phi}{2C_\gamma}\right)^{\frac{1}{4-m}}\phi^{\frac{1-m}{4-m}}H^{\frac{11-3m}{4-m}}(-\dot{H})^{-\frac{(3-m)}{4-m}} \times [1-\epsilon(\alpha+\beta H^2)^{-1/2}]^{-\frac{(3-m)}{4-m}}. \quad (19)$$

Also, the power spectrum as function of the field ϕ, from Eqs. (16) and (19) can be written as

$$\mathcal{P}_\mathcal{R}(\phi) \simeq k_1\phi^{\frac{1-m}{4-m}}(F^{-1}[K\phi])^{\frac{2f(4-m)+m-5}{4-m}} \times \left[1-\epsilon\left(\alpha+\frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}}\right)^{-1/2}\right]^{-\frac{(3-m)}{4-m}}, \quad (20)$$

where the constant k_1 is defined as $k_1 = \frac{\sqrt{3}\pi}{4\mu^2}\left(\frac{\mu^2 C_\phi}{2C_\gamma}\right)^{\frac{1}{4-m}} \times (Af)^{\frac{8-2m}{4-m}}(1-f)^{\frac{m-3}{4-m}}$.

Likewise, the power spectrum as function of the number N, yields

$$\mathcal{P}_\mathcal{R}(N) = k_2(F[J(N)])^{\frac{1-m}{4-m}}(J[N])^{\frac{2f(4-m)+m-5}{4-m}} \times \left[1-\epsilon\left(\alpha+\frac{\beta A^2 f^2}{(J[N])^{2(1-f)}}\right)^{-1/2}\right]^{-\frac{(3-m)}{4-m}}, \quad (21)$$

where $J(N)$ and k_2 are given by $J(N) = [\frac{1+f(N-1)}{Af}]^{\frac{1}{f}}$ and $k_2 = k_1 K^{-\frac{1-m}{4-m}}$, respectively.

The scalar spectral index n_s given by $n_s - 1 = \frac{d\ln \mathcal{P}_\mathcal{R}}{d\ln k}$, where the wave number k, is associated with the number of e-folds through $d\ln k(\phi) = dN(\phi) = (H/\dot{\phi})d\phi$. By using Eqs. (2) and (21), this yields

$$n_s = 1 - \frac{5-m-2f(4-m)}{Af(4-m)(F^{-1}[K\phi])^f} + n_2 + n_3, \quad (22)$$

where n_2 and n_3 are given by

$$n_2 = \mu\frac{1-m}{4-m}\sqrt{\frac{2(1-f)}{Af}}\frac{(F^{-1}[K\phi])^{-f/2}}{\phi} \times \left[1-\epsilon\left(\alpha+\frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}}\right)^{-1/2}\right]^{1/2},$$

and

$$n_3 = \epsilon\frac{\beta Af(1-f)}{(F^{-1}[K\phi])^{2-f}}\left(\alpha+\frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}}\right)^{-3/2} \times \left[1-\epsilon\left(\alpha+\frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}}\right)^{-1/2}\right]^{-1},$$

respectively.

Also, n_s can be written in terms of N. By using Eqs. (18) and (22), we obtain

$$n_s = 1 - \frac{5-m-2f(4-m)}{(4-m)[1+f(N-1)]} + n_{2N} + n_{3N}, \quad (23)$$

where

$$n_{2N} = \mu K\frac{1-m}{4-m}\sqrt{\frac{2(1-f)}{Af}}\frac{(J[N])^{-f/2}}{F[J(N)]} \times \left[1-\epsilon\left(\alpha+\frac{\beta A^2 f^2}{(J[N])^{2(1-f)}}\right)^{-1/2}\right]^{1/2},$$

and

$$n_{3N} = \epsilon\frac{\beta Af(1-f)}{(J[N])^{2-f}}\left(\alpha+\frac{\beta A^2 f^2}{(J[N])^{2(1-f)}}\right)^{-3/2} \times \left[1-\epsilon\left(\alpha+\frac{\beta A^2 f^2}{(J[N])^{2(1-f)}}\right)^{-1/2}\right]^{-1}.$$

We observe numerically from Eq. (23) that the value of $n_s \gg 1$, for the positive value of ϵ, i.e., the positive branch. This value of scalar spectral index is disproved by the observational data. In this way, the model of warm intermediate inflation on a warped DGP does not work for the case $\epsilon = +1$. In the following, we will study the negative branch in which $\epsilon = -1$.

On the other hand, it is well known that the generation of tensor perturbations during the inflationary epoch would produce gravitational waves [39]. In the warped DGP model, the amplitude of gravitational waves [15], becomes

$$\mathcal{P}_g = \frac{64\pi}{m_p^2}(H/2\pi)^2 G_\gamma^2(x), \quad (24)$$

where $G_\gamma^{-2}(x) = \gamma + (1-\gamma)F(x)^{-2}$ is the correction to standard general relativity. Here, the parameter $\gamma = (\mu/m_p)^2$ and the function $F(x) = [\sqrt{1+x} - x^2\sinh^{-1}(1/x)]^{-1/2}$, in which $x = H/\bar{\mu}$, where $\bar{\mu}$ is the energy scale related with the bulk curvature [40]. In particular, in the case when $x \to 0$, then $G_\gamma \to 1$ and thus reduces to standard amplitude of gravitational waves, where $\mathcal{P}_g = \frac{64\pi}{m_p^2}(H/2\pi)^2$. Also, in the case when $\gamma \to 0$, the expression for $\mathcal{P}_g$, coincides with the amplitude of gravitational waves in the RS case [41].

In this form, using Eqs. (20) and (24), we may define the tensor-to-scalar ratio as $r = (\mathcal{P}_g/\mathcal{P}_\mathcal{R})$, and in terms of the scalar field, this ratio, in the weak dissipative regime, can be written as

$$r(\phi) \simeq \frac{16A^2 f^2}{\pi m_p^2 (F^{-1}[K\phi])^{2(1-f)}}\left[\frac{G_\gamma^2(\phi)}{\mathcal{P}_\mathcal{R}(\phi)}\right]. \quad (25)$$

Also, the tensor-to-scalar ratio can be rewritten in terms of the number of e-folds N, as

$$r(N) \simeq \frac{16A^2 f^2}{\pi m_p^2 (J[N])^{2(1-f)}}\left[\frac{G_\gamma^2(N)}{\mathcal{P}_\mathcal{R}(N)}\right]. \quad (26)$$

In Fig. 1 we show the evolution of the ratio $\Gamma/3H$ and the tensor-to-scalar ratio r on the scalar spectral index n_s in the weak dissipative regime, for the specific case $m = 3$, in which the dissipation coefficient becomes $\Gamma = C_\phi T^3/\phi^2$. In both panels we have considered three different values of C_ϕ. The upper panel shows the dependence of $\Gamma/3H$ on the warm inflation and we confirm that the model remains in the weak dissipative regime ($\Gamma/3H < 1$) during inflation. In the lower panel, we show the two-dimensional marginalized constraints, at 68% and 95% levels of confidence, for

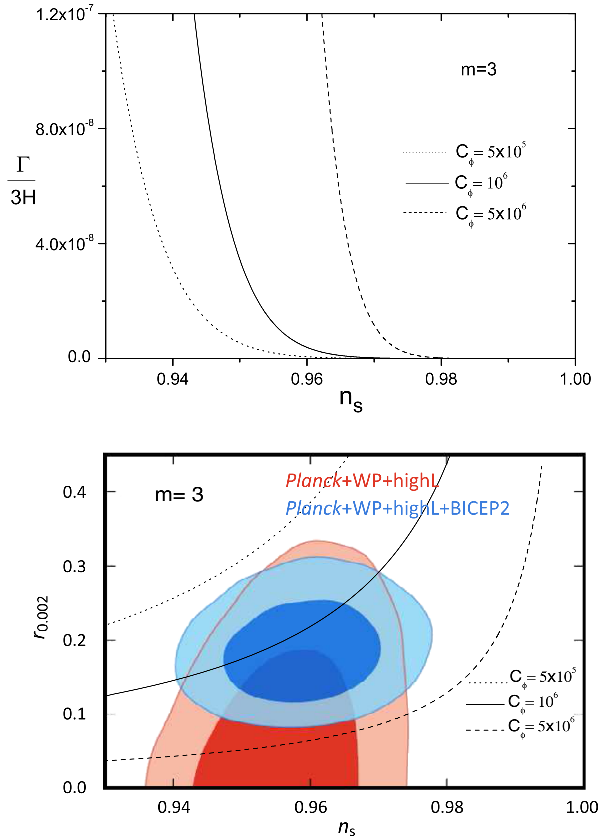

FIG. 1 (color online). The evolution of the ratio $\Gamma/3H$ versus the scalar spectrum index n_s (upper panel) and the evolution of the tensor-scalar ratio r versus the scalar spectrum index n_s (lower panel) in the weak dissipative regime, for three different values of the parameter C_ϕ and the specific case $m = 3$, i.e., $\Gamma \propto T^3/\phi^2$. In both panels, the dotted, solid, and dashed lines correspond to the pairs ($A = 0.08$, $f = 0.43$), ($A = 0.19$, $f = 0.42$) and ($A = 0.23$, $f = 0.39$), respectively. Also, in these plots we have taken the values $C_\gamma = 70$, $m_p = 1$, $\mu = 0.99$, $\eta = 1$, $\epsilon = -1$, $\mathcal{A}_0 = 1$ and $\Omega_{rc} = (\beta H_0^2)^{-1} = 0.014$ [22].

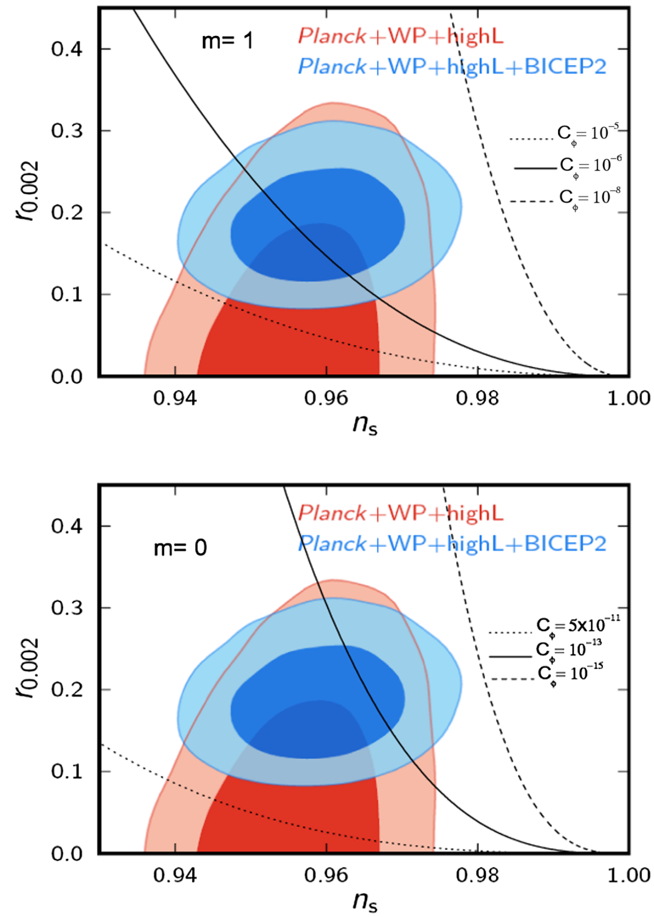

FIG. 2 (color online). The upper and lower panels show the evolution of the tensor-scalar ratio r versus the scalar spectrum index n_s in the weak dissipative regime, for the cases $m = 1$ and $m = 0$, respectively. In both panels we consider three different values of the parameter C_ϕ. For the case $m = 1$ (upper panel), the dotted, solid, and dashed lines are for the pairs ($A = 0.67$, $f = 0.29$), ($A = 0.79$, $f = 0.29$) and ($A = 0.95$, $f = 0.29$). For the case $m = 0$ (lower panel), the dotted, solid, and dashed lines are for the pairs ($A = 0.99$, $f = 0.27$), ($A = 1.38$, $f = 0.27$) and ($A = 1.75$, $f = 0.26$). Also, in both panels we have taken the values $C_\gamma = 70$, $m_p = 1$, $\mu = 0.99$, $\eta = 1$, $\epsilon = -1$, $\mathcal{A}_0 = 1$ and $\Omega_{rc} = (\beta H_0^2)^{-1} = 0.014$ [22].

the tensor-to-scalar ratio and the scalar spectral index (taken BICEP2 experiment data in connection with Planck + WP + highL, see Ref. [28]). From Eqs. (2), (15), (16), and (18) we can obtain in the weak dissipative regime $R < 1$, the ratio $\Gamma/3H$ as a function of the number of e-folds N, i.e., $\Gamma/3H = f(N)$, and together with Eq. (23), we numerically obtain the parametric plot of the curve $\Gamma/3H = \Gamma/3H(n_s)$ (upper panel). Analogously, we consider Eqs. (23) and (26) and we numerically find the parametric plot of the consistency relation $r = r(n_s)$ (lower panel). Here, we consider that the range for the number of e-folds N is given by $30 \leq N \leq 120$. In these plots we use the values of $C_\gamma = 70$, $m_p = 1$, $\mu = 0.99$, $\eta = 1$, $\epsilon = -1$, $\mathcal{A}_0 = 1$ and $\Omega_{rc} = (\beta H_0^2)^{-1} = 0.014$ [22], in which $H_0 = 68.6\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. In addition, we numerically make use of Eqs. (21) and (23), and we find that $A = 0.08 \times 10^{-6}$ and $f = 0.43$ for the value of $C_\phi = 5 \times 10^5$ for which $n_s = 0.96$, $\mathcal{P}_\mathcal{R} = 2.43 \times 10^{-9}$ and $N = 60$. Similarly, $C_\phi = 10^6$ corresponds to $A = 0.19$, $f = 0.42$; for the case in which $C_\phi = 5 \times 10^6$, then $A = 0.23$, $f = 0.39$. From the lower plot we find that the range for the parameter C_ϕ, in the special case $m = 3$, is given by $5 \times 10^5 < C_\phi < 5 \times 10^6$, which is well corroborated from the BICEP2 experiment and also from Planck data.

In Fig. 2 we show the evolution of the tensor-to-scalar ratio r on the scalar spectral index n_s for the weak dissipative regime, where in the upper panel we fix $m = 1$ and in the lower panel $m = 0$. As before we consider three values of C_ϕ. Again, we use Eqs. (23) and (26) and we numerically find the parametric plot $r = r(n_s)$ for $m = 1$ and $m = 0$, where $C_\gamma = 70$, $m_p = 1$, $\mu = 0.99$, $\eta = 1$, $\epsilon = -1$, $\mathcal{A}_0 = 1$ and $\Omega_{rc} = (\beta H_0^2)^{-1} = 0.014$ [22]. As before, we solve Eqs. (21) and (23), and we find that for the case $m = 1$ (upper panel), the dotted, solid, and dashed lines correspond to the pairs ($A = 0.67$, $f = 0.29$), ($A = 0.79$, $f = 0.29$) and ($A = 0.95$, $f = 0.29$), respectively. For the case $m = 0$ (lower panel), the dotted, solid, and dashed lines correspond to the pairs ($A = 0.99$, $f = 0.27$), ($A = 1.38$, $f = 0.27$) and ($A = 1.75$, $f = 0.26$), respectively. From the upper plot we obtain the range for C_ϕ in the specific case $m = 1$, given by $10^{-8} < C_\phi < 10^{-5}$. From the lower plot we observe that the range for C_ϕ in the case $m = 0$ is given by $10^{-15} < C_\phi < 5 \times 10^{-11}$. These ranges of the parameter C_ϕ, for both models, are well supported by BICEP2 experiment data and Planck data. Finally, for the special case $m = -1$, i.e., $\Gamma \propto \phi^2/T^1$, we find that the range for C_ϕ becomes $10^{-22} < C_\phi < 10^{-16}$, where $C_\phi = 10^{-22}$ corresponds to $A = 2.22$, $f = 0.25$ and for the case in which $C_\phi = 10^{-16}$, then $A = 1.27$, $f = 0.24$ (not shown). In this form, we observe that when we decrease the value of the parameter m the range of the parameter C_ϕ also decreases. Also, we noted that in the weak dissipative regime the constraint on the parameter C_ϕ only arises from the BICEP2 experiment (or Planck data) and not from of the ratio $\Gamma/3H$.

In Fig. 3 we show the evolution of the ratio T/H on the number of e-folds N for the weak dissipative regime, where in the upper panel we fix $m = 3$ and in the lower panel $m = 1$. In order to write down values for the ratio T/H and the number of e-folds N, for the special cases $m = 3$ and $m = 1$, we utilize Eqs. (2), (12), (16), and (18), together with the same parameters from Figs. 1 and 2. From the upper panel we note that the value $C_\phi > 5 \times 10^5$ is well supported by the necessary condition for the warm inflation scenario, i.e., $T > H$. In particular, for the value $C_\phi = 10^6$ and evaluating for the value of $N = 60$ (where $n_s = 0.96$), we get that the value of the ratio $T/H \simeq 1.47$, for the value $C_\phi = 5 \times 10^6$ corresponds to $T/H \simeq 5.51$ and for $C_\phi = 5 \times 10^5$ it corresponds to $T/H \simeq 0.79$. In this way, we find that the range for the parameter C_ϕ, in the special case $m = 3$, is given by $5 \times 10^5 < C_\phi < 5 \times 10^6$, which are well corroborated from Planck data and the BICEP2 experiment together with the condition for warm inflation $T/H > 1$.

From the lower panel we observe that the value $C_\phi \gtrsim 10^{-6}$ is well supported by the condition for warm inflation, i.e., $T/H > 1$ (for $N \geq 60$). In particular, for the value $C_\phi = 10^{-5}$, we obtain that $T/H \simeq 5.28$ for $N = 60$ and for the value $C_\phi = 10^{-6}$ corresponds to $T/H \simeq 1.67$. For the values $C_\phi = 10^{-8}$ and $N = 60$ corresponds to $T/H \simeq 0.17$ and the model of the weak dissipative regime is disfavored from the essential condition for the warm inflation scenario, since the ratio $T/H < 1$. It interesting to note that from the condition $T/H > 1$, we have found a lower bound for the parameter C_ϕ. In this form, for the value $m = 1$ we can set a new constraint for the parameter C_ϕ, given by $10^{-6} \lesssim C_\phi < 10^{-5}$ from BICEP2 experiment (or Planck data) and the condition $T/H > 1$.

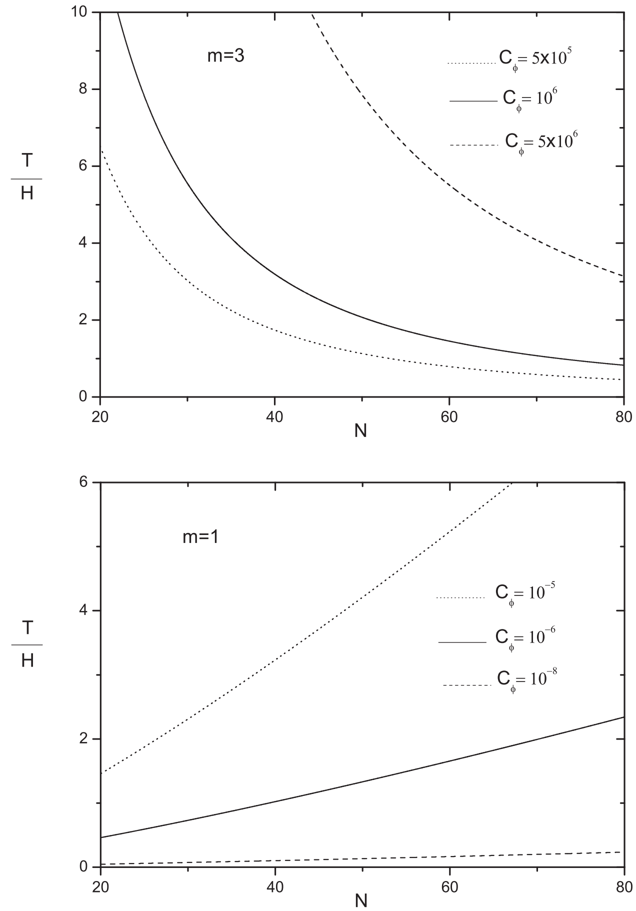

FIG. 3. The upper and lower panels show the evolution of the ratio T/H versus the number of e-folds N in the weak dissipative regime, for the cases $m = 3$ and $m = 1$, respectively. Here, we have used the same parameters of the Figs. 1 and 2.

For the case $m = 0$ the evolution of the ratio T/H on the number of e-folds N for the weak dissipative regime, is similar to the case $m = 1$ (figure not shown). As before, we note that the value $C_\phi \gtrsim 10^{-13}$ is well supported by the condition for the warm inflation scenario $T/H > 1$. Also, we find that for the values $C_\phi = 10^{-15}$ and $N = 60$ corresponds to $T/H \simeq 0.15$ and then $T/H < 1$. Again, we observe that from the condition $T/H > 1$ we have obtained a lower bound for C_ϕ, and the range for this parameter from BICEP2 experiment (or Planck data) and $T/H > 1$, is given by $10^{-6} \lesssim C_\phi < 10^{-5}$. Finally, for the special case $m = -1$, we note that the value $C_\phi \gtrsim 10^{-19}$ is well supported by the condition $T/H > 1$ for $N \geq 60$ (not shown). Also, in particular we note that for the values $C_\phi = 10^{-22}$ and $N = 60$ the model of the weak dissipative regime is disfavored from the condition for the warm inflation scenario, since the rate $T/H \simeq 0.14$. Again, we observe that from $T/H > 1$ we have found a lower bound for C_ϕ. In this form, for $m = -1$ the new constraint for the parameter C_ϕ from the Planck data (or BICEP2) and $T/H > 1$, is given by $10^{-19} \lesssim C_\phi < 10^{-16}$. Finally, we note that in the weak dissipative regime from the essential condition for warm inflation $T/H > 1$, we have found a lower bound for the parameter C_ϕ and this bound becomes independent from Planck data and BICEP2 experiment.

B. The strong dissipative regime

Now we study the case of the strong dissipative regime, i.e., $\Gamma > 3H$. By using Eqs. (9) and (15), and considering the intermediate expansion given by Eq. (2), we obtain a relation between the scalar field and cosmological time. However, we must study two cases separately, namely $m = 3$, and $m \neq 3$. For the case $m = 3$, the solution for the scalar field yields

$$\phi(t) - \phi_0 = \exp\left[\frac{\tilde{F}[t]}{\tilde{K}}\right], \tag{27}$$

again $\phi(t = 0) = \phi_0$ is an integration constant, the constant $\tilde{K}$ is defined by

$$\tilde{K} \equiv \left(\frac{C_\phi}{6\mu^2}\right)^{\frac{1}{2}}\left(\frac{3\mu^2}{2C_\gamma}\right)^{\frac{3}{8}}(1-f)^{\frac{7}{8}}\beta^{-\frac{(5f+2)}{16(1-f)}}(Af)^{-\frac{7}{8(1-f)}},$$

and the new function $\tilde{F}[t]$ corresponds to the incomplete Lauricella function [34] given by

$$\tilde{F}[t] \equiv \frac{[\epsilon(\alpha + \frac{\beta}{t^{2(1-f)}})^{-1/2}]^{2(\nu-1)}}{2(\nu-1)} F_D^{(3)}\left[2(\nu-1); \nu, \nu, -\frac{1}{8}, 2\nu-1, \sqrt{\alpha}, -\sqrt{\alpha}, \epsilon\left(\alpha + \frac{\beta}{t^{2(1-f)}}\right)^{-1/2}\right],$$

where the constant ν is defined as $\nu = \frac{18-11f}{16(1-f)}$.

For the case $m \neq 3$, the solution for the scalar field can be written as

$$\varphi(t) - \varphi_0 = \frac{\tilde{F}_m[t]}{\tilde{K}_m}, \tag{28}$$

where now the new scalar field φ is defined as $\varphi(t) = \frac{2}{3-m}\phi(t)^{\frac{3-m}{2}}$, the constant $\tilde{K}_m = (\frac{C_\phi}{6\mu^2})^{\frac{1}{2}}(\frac{3\mu^2}{2C_\gamma})^{\frac{m}{8}}(1-f)^{\frac{4+m}{8}}\beta^{\frac{4+f(m-8)-2m}{16(1-f)}} \times (Af)^{-\frac{4+m}{8(1-f)}}$, and the function $\tilde{F}_m[t]$ is defined as

$$\tilde{F}_m[t] \equiv \frac{[\epsilon(\alpha + \frac{\beta}{t^{2(1-f)}})^{-1/2}]^{2(\nu-1)}}{2(\nu-1)} F_D^{(3)}\left[2(\nu-1); \nu, \nu, -\frac{(4-m)}{8}, 2\nu-1, \sqrt{\alpha}, -\sqrt{\alpha}, \epsilon\left(\alpha + \frac{\beta}{t^{2(1-f)}}\right)^{-1/2}\right],$$

in which the constant $\nu_m = \frac{2(6+m)-f(8+m)}{16(1-f)}$.

In this regime, the Hubble parameter as a function of the inflaton field ϕ for both cases becomes

$$H(\phi) = \frac{Af}{(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{1-f}}, \quad \text{for } m = 3, \tag{29}$$

and

$$H(\phi) = \frac{Af}{(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{1-f}}, \quad \text{for } m \neq 3. \tag{30}$$

As before, considering the slow-roll approximation, the scalar potential in the strong dissipative regime from Eq. (14) yields

$$\begin{aligned} V(\phi) \simeq& \frac{\eta\rho_0}{2}\left(\alpha + \frac{\beta A^2 f^2}{(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{2(1-f)}}\right)\left[1 - \epsilon\left(\alpha + \frac{\beta A^2 f^2}{(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{2(1-f)}}\right)^{-1/2}\right]^2 - \frac{\mathcal{A}_0^2\rho_0}{2\eta} \\ &-\frac{3}{2}\mu^2\frac{Af(1-f)}{(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{2-f}}\left[1 - \epsilon\left(\alpha + \frac{\beta A^2 f^2}{(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{2(1-f)}}\right)^{-1/2}\right] \end{aligned} \tag{31}$$

for the case in which $m = 3$, and

$$V(\phi) \simeq \frac{\eta\rho_0}{2}\left(\alpha + \frac{\beta A^2 f^2}{(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{2(1-f)}}\right)\left[1 - \epsilon\left(\alpha + \frac{\beta A^2 f^2}{(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{2(1-f)}}\right)^{-1/2}\right]^2 - \frac{\mathcal{A}_0^2\rho_0}{2\eta}$$
$$- \frac{3}{2}\mu^2 \frac{Af(1-f)}{(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{2-f}}\left[1 - \epsilon\left(\alpha + \frac{\beta A^2 f^2}{(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{2(1-f)}}\right)^{-1/2}\right] \quad (32)$$

for $m \neq 3$.

Also, the dissipation coefficient from Eqs. (15), (27), and (28) becomes

$$\Gamma(\phi) = \delta\phi^{-2}(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{-\frac{3(2-f)}{4}}[1 - \epsilon(\alpha + \beta H^2)^{-1/2}]^{3/4},$$
$$\text{for } m = 3, \quad (33)$$

where the constant $\delta = C_\phi(\frac{3\mu^2 Af(1-f)}{2C_\gamma})^{3/4}$ and

$$\Gamma(\phi) = \delta_m\phi^{1-m}(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{-\frac{m(2-f)}{4}}[1 - \epsilon(\alpha + \beta H^2)^{-1/2}]^{m/4}$$
$$\text{for } m \neq 3. \quad (34)$$

Here the constant δ_m is defined as $\delta_m = C_\phi(\frac{3\mu^2 Af(1-f)}{2C_\gamma})^{m/4}$.

Now for this regime, the dimensionless slow-roll parameter ε is given by $\varepsilon = -\frac{\dot{H}}{H^2} = \frac{1-f}{Af(\tilde{F}^{-1}[\tilde{K}\ln\phi])^f}$, for the case in which $m = 3$; for the case $m \neq 3$, we get $\varepsilon = \frac{1-f}{Af(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^f}$.

As before, the condition for inflation $\ddot{a} > 0$ is satisfied when the scalar field $\phi > \exp[\frac{1}{\tilde{K}}\tilde{F}[(\frac{1-f}{Af})^{1/f}]]$, for the case in which $m = 3$; for $m \neq 3$, the condition is satisfied for the new scalar field when $\varphi > \frac{1}{\tilde{K}_m}\tilde{F}_m[(\frac{1-f}{Af})^{1/f}]$. As in the weak dissipative regime, the inflationary scenario begins at the earliest possible scenario in which $\varepsilon = 1$. Here, $\phi_1 = \exp[\frac{1}{\tilde{K}}\tilde{F}[(\frac{1-f}{Af})^{1/f}]]$, for the case in which $m = 3$, and $\varphi_1 = \frac{1}{\tilde{K}_m}\tilde{F}_m[(\frac{1-f}{Af})^{1/f}]$ for the case $m \neq 3$.

In the strong dissipative regime, the expression for the number of e-folds between two different values ϕ_1 and ϕ_2, from Eqs. (2), (27), and (28), becomes

$$N = \int_{t_1}^{t_2} H dt = A[(\tilde{F}^{-1}[\tilde{K}\ln\phi_2])^f - (\tilde{F}^{-1}[\tilde{K}\ln\phi_1])^f],$$
$$\text{for } m = 3, \quad (35)$$

and

$$N = A[(\tilde{F}_m^{-1}[\tilde{K}_m\varphi_2])^f - (\tilde{F}_m^{-1}[\tilde{K}_m\varphi_1])^f], \quad \text{for } m \neq 3. \quad (36)$$

On the other hand, as before the power spectrum related to the curvature spectrum could be written as $\mathcal{P}_\mathcal{R} \simeq \frac{H}{\dot{\phi}}\delta\phi$, where in the strong dissipative regime, i.e., $\Gamma > 3H$, we get that $\delta\phi^2 \simeq \frac{k_F T}{2\pi^2}$ [1], in which $k_F = \sqrt{\Gamma H}$. In this way, from Eqs. (2), (11), and (15), the expression for the spectrum of the scalar perturbation yields

$$P_\mathcal{R} \simeq \frac{H^{\frac{5}{2}}\Gamma^{\frac{1}{2}}T}{2\pi^2\dot{\phi}^2} = \frac{C_\phi^{3/2}}{2\pi^2}\left(\frac{1}{6\mu^2}\right)\left(\frac{3\mu^2}{2C_\gamma}\right)^{\frac{3m+2}{8}}\phi^{\frac{3(1-m)}{2}}H^{\frac{3}{2}}(-\dot{H})^{\frac{3m-6}{8}}$$
$$\times [1 - \epsilon(\alpha + \beta H^2)^{-1/2}]^{\frac{3m-6}{8}}. \quad (37)$$

As before, it is necessary to separate the specific cases $m = 3$ and $m \neq 3$. Replacing Eqs. (2), (27), and (28) in Eq. (37), we can obtain the power spectrum in terms of the scalar field for both values of m. In this form, for the case in which $m = 3$, we get

$$\mathcal{P}_\mathcal{R} = k\phi^{-3}(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{\frac{3(5f-6)}{8}}$$
$$\times [1 - \epsilon(\alpha + \beta A^2 f^2(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{-2(1-f)})^{-1/2}]^{\frac{3}{8}}, \quad (38)$$

where the constant k is defined as $k = \frac{C_\phi^{3/2}}{2\pi^2}\frac{1}{6\mu^2}(\frac{3\mu^2}{2C_\gamma})^{\frac{11}{8}} \times (1-f)^{\frac{3}{8}}(Af)^{\frac{15}{8}}$. For the specific case in which $m \neq 3$, the spectrum of the scalar perturbation yields

$$\mathcal{P}_\mathcal{R} = k_m\phi^{\frac{3(1-m)}{2}}(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{\frac{3[f(m+2)-2m]}{8}}$$
$$\times [1 - \epsilon(\alpha + \beta A^2 f^2(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{-2(1-f)})^{-1/2}]^{\frac{3m-6}{8}}, \quad (39)$$

where the constant $k_m = \frac{C_\phi^{3/2}}{2\pi^2}\frac{1}{6\mu^2}(\frac{3\mu^2}{2C_\gamma})^{\frac{3m+2}{8}}(1-f)^{\frac{3m-6}{8}}(Af)^{\frac{3m+6}{8}}$.

In order to manipulate numerically the equations, it is useful to rewrite the scalar power spectrum in terms of the number of e-folds. By using Eqs. (35) and (36), the above expressions becomes

$$\mathcal{P}_\mathcal{R} = k\exp\left(-\frac{3}{\tilde{K}}\tilde{F}[J[N]]\right)(J[N])^{\frac{3(5f-6)}{8}}$$
$$\times [1 - \epsilon(\alpha + \beta A^2 f^2(J[N])^{-2(1-f)})^{-1/2}]^{\frac{3}{8}}, \quad (40)$$

for the case in which $m = 3$, and

$$\mathcal{P}_\mathcal{R} = \tilde{\gamma}_m(\tilde{F}_m[J[N]])^{\frac{3(1-m)}{3-m}}(J[N])^{\frac{3[f(m+2)-2m]}{8}}$$
$$\times [1 - \epsilon(\alpha + \beta A^2 f^2(J[N])^{-2(1-f)})^{-1/2}]^{\frac{3m-6}{8}} \quad (41)$$

for the case $m \neq 3$. Here, the constant $\tilde{\gamma}_m$ is given by $\tilde{\gamma}_m = k_m(\frac{2\tilde{K}_m}{3-m})^{-\frac{3(1-m)}{3-m}}$.

From Eqs. (38) and (39), the scalar spectral index n_s in the strong dissipative regime yields

$$n_s \simeq 1 + \frac{3(5f-6)}{8Af}(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{-f} + n_{\epsilon 1} + n_{\epsilon 2}, \quad (42)$$

where $n_{\epsilon 1}$ and $n_{\epsilon 2}$, are given by

$$n_{\epsilon 1} = -3\tilde{K}(1-f)^{1/8}(Af)^{-3/8}(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{\frac{2-3f}{8}} \times [1-\epsilon(\alpha+\beta A^2 f^2(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{-2(1-f)})^{-1/2}]^{1/8},$$

and

$$n_{\epsilon 2} = -\epsilon\frac{3}{8}\beta Af(1-f)(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{f-2} \times (\alpha+\beta A^2 f^2(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{-2(1-f)})^{-3/2} \times [1-\epsilon(\alpha+\beta A^2 f^2(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{-2(1-f)})^{-1/2}]^{-1},$$

for the specific case in which $m=3$. For the case $m \neq 3$ we have

$$n_s \simeq 1 + \frac{3[f(2+m)-2m]}{8Af}(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{-f} + n_{\epsilon 1_m} + n_{\epsilon 2_m}, \quad (43)$$

where

$$n_{\epsilon 1_m} = \frac{3(1-m)}{2} \times \tilde{K}_m(1-f)^{\frac{4-m}{8}}(Af)^{-\frac{m}{8}}\phi^{\frac{m-3}{2}}(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{\frac{[m(2-f)-4]}{8}} \times [1-\epsilon(\alpha+\beta A^2 f^2(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{-2(1-f)})^{-1/2}]^{\frac{4-m}{8}},$$

and

$$n_{\epsilon 2_m} = -\epsilon\frac{(3m-6)}{8}\beta Af(1-f)(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{f-2} \times (\alpha+\beta A^2 f^2(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{-2(1-f)})^{-3/2} \times [1-\epsilon(\alpha+\beta A^2 f^2(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{-2(1-f)})^{-1/2}]^{-1}.$$

Analogously as before, the scalar spectral index n_s can be rewritten in terms of the number of e-folds. By considering Eqs. (35) and (36), the scalar spectral index results

$$n_s \simeq 1 + \frac{3(5f-6)}{8Af}(J[N])^{-f} + n_{\epsilon 1}[J[N]] + n_{\epsilon 2}[J[N]], \quad (44)$$

where

$$n_{\epsilon 1}[J[N]] = -3\tilde{K}(1-f)^{1/8}(Af)^{-3/8}(J[N])^{\frac{2-3f}{8}} \times [1-\epsilon(\alpha+\beta A^2 f^2(J[N])^{-2(1-f)})^{-1/2}]^{1/8},$$

and

$$n_{\epsilon 2}[J[N]] = -\epsilon\frac{3}{8}\beta Af(1-f)(J[N])^{f-2} \times (\alpha+\beta A^2 f^2(J[N])^{-2(1-f)})^{-3/2} \times [1-\epsilon(\alpha+\beta A^2 f^2(J[N])^{-2(1-f)})^{-1/2}]^{-1}$$

for the specific case in which $m=3$. For the case $m \neq 3$ the spectral index can be written as

$$n_s \simeq 1 + \frac{3[f(2+m)-2m]}{8Af}(J[N])^{-f} + n_{\epsilon 1_m}[J[N]] + n_{\epsilon 2_m}J[[N]], \quad (45)$$

where

$$n_{\epsilon 1_m}[J[N]] = \frac{3(1-m)}{3-m}(1-f)(Af)^{\frac{[(f-2)m-4]}{8(1-f)}}\beta^{\frac{[4+f(m-8)-2m]}{16(1-f)}} \times (\tilde{F}_m[J[N]])^{-1}(J[N])^{\frac{[m(2-f)-4]}{8}} \times [1-\epsilon(\alpha+\beta A^2 f^2(J[N])^{-2(1-f)})^{-1/2}]^{\frac{4-m}{8}},$$

and

$$n_{\epsilon 2_m}[J[N]] = -\epsilon\frac{(3m-6)}{8}\beta Af(1-f)(J[N])^{f-2} \times (\alpha+\beta A^2 f^2(J[N])^{-2(1-f)})^{-3/2} \times [1-\epsilon(\alpha+\beta A^2 f^2(J[N])^{-2(1-f)})^{-1/2}]^{-1}.$$

On the other hand, from Eqs. (24) and (39), the tensor-to-scalar ratio for the warped DGP model in the strong dissipative regime, for the case $m=3$, can be written as

$$r = \zeta\phi^3(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{\frac{f+2}{8}} \times [1-\epsilon(\alpha+\beta A^2 f^2(\tilde{F}^{-1}[\tilde{K}\ln\phi])^{-2(1-f)})^{-1/2}]^{-\frac{3}{8}}, \quad (46)$$

where the constant $\zeta = 24(\frac{\mu^2}{m_p^2})\frac{(Af)^{\frac{1}{8}}}{C_\phi^{3/2}}(\frac{2C_\gamma}{3\mu^2})^{\frac{11}{8}}(1-f)^{-\frac{3}{8}}$, and for the specific case in which $m \neq 3$, we get

$$r = \zeta_m\phi^{-\frac{3}{2}(1-m)}(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{\frac{1}{8}[-16+f(10-3m)+6m]} \times [1-\epsilon(\alpha+\beta A^2 f^2(\tilde{F}_m^{-1}[\tilde{K}_m\varphi])^{-2(1-f)})^{-1/2}]^{-\frac{1}{8}(3m-6)}, \quad (47)$$

where $\zeta_m = 24(\frac{\mu^2}{m_p^2})\frac{(Af)^{\frac{1}{8}(10-3m)}}{C_\phi^{3/2}}(\frac{2C_\gamma}{3\mu^2})^{\frac{3m+2}{8}}(1-f)^{-\frac{(3m-6)}{8}}$.

Analogously as before, the tensor-to-scalar ratio r as a function of the number e-folds N becomes

$$r(N) = \zeta\exp\left(\frac{3}{\tilde{K}}\tilde{F}[J[N]]\right)(J[N])^{\frac{f+2}{8}} \times [1-\epsilon(\alpha+\beta A^2 f^2(J[N])^{-2(1-f)})^{-1/2}]^{-\frac{3}{8}}, \quad (48)$$

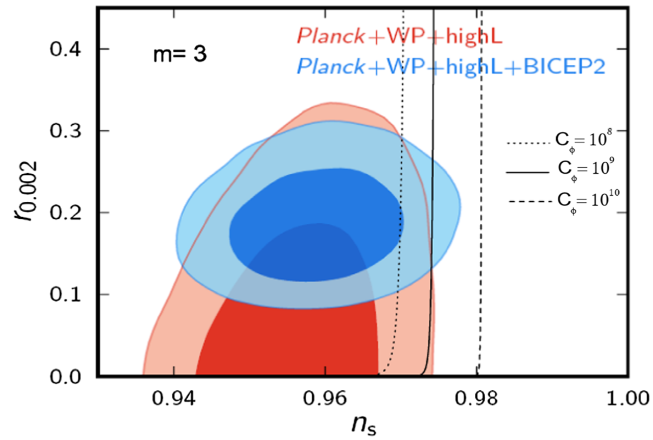

FIG. 4 (color online). Evolution of the tensor-scalar ratio r versus the scalar spectrum index n_s in the strong dissipative regime, for three different values of the parameter C_ϕ and $m = 3$. The dotted, solid , and dashed lines are for the pairs ($A = 4.86 \times 10^{-5}$, $f = 0.64$), ($A = 3.92 \times 10^{-6}$, $f = 0.82$) and ($A = 1.08 \times 10^{-6}$, $f = 0.99$), respectively. Also, in this plot we have taken the values $C_\gamma = 70$, $m_p = 1$, $\mu = 0.99$, $\eta = 1$, $\epsilon = -1$, $\mathcal{A}_0 = 1$ and $\Omega_{rc} = (\beta H_0^2)^{-1} = 0.014$ [22].

for the case in which $m = 3$, and

$$r = \tilde{\zeta}_m(\tilde{F}_m[J[N]])^{-\frac{3(1-m)}{3-m}}(J[N])^{\frac{1}{8}[-16+f(10-3m)+6m]} \times [1 - \epsilon(\alpha + \beta A^2 f^2 (J[N])^{-2(1-f)})^{-1/2}]^{-\frac{1}{8}(3m-6)}, \quad (49)$$

for the case $m \neq 3$, where the constant $\tilde{\zeta}_m$ is defined as $\tilde{\zeta}_m = \zeta_m(\frac{2\tilde{K}_m}{3-m})^{\frac{3(1-m)}{3-m}}$.

In Fig. 4 we establish the dependence of the tensor-scalar ratio r versus the primordial tilt n_s, for the specific case in which we fix $m = 3$ ($\Gamma \propto T^3/\phi^2$), in the strong dissipative regime. Here, we have considered three different values of the parameter C_ϕ. Again, we show the two-dimensional marginalized constraints (68% and 95% C.L.) from BICEP2 experiment [28]. In order to write down values for the tensor-scalar ratio and the scalar spectrum index for the special case $m = 3$, i.e., $\Gamma \propto T^3/\phi^2$, we use Eqs. (44) and (48), where $C_\gamma = 70$, $m_p = 1$, $\mu = 0.99$, $\eta = 1$, $\epsilon = -1$, $\mathcal{A}_0 = 1$ and $\Omega_{rc} = (\beta H_0^2)^{-1} = 0.014$ [22]. Also, we numerically make use of Eqs. (40) and (44) and obtain $A = 1.08 \times 10^{-6}$ and $f = 0.99$ for the value of $C_\phi = 10^8$ for which $n_s = 0.96$, $\mathcal{P}_\mathcal{R} = 2.43 \times 10^{-9}$ and $N = 60$. Similarly, $C_\phi = 10^9$ corresponds to $A = 3.92 \times 10^{-6}$, $f = 0.82$; for the case in which $C_\phi = 10^{10}$, then $A = 4.86 \times 10^{-5}$, $f = 0.64$. From the plot we observe that the value $C_\phi < 10^{10}$ is well corroborated by the C.L. from the BICEP2 experiment and also from Planck data. Additionally, we observe that the parameter $C_\phi > 10^8$ is well supported by the strong regime, in which $\Gamma/3H > 1$ (not shown). In this form, the range for the parameter C_ϕ in the specific case in which $m = 3$, is given by $10^8 < C_\phi < 10^{10}$. We note that this range for the parameter C_ϕ in the strong dissipative regime, becomes similar to the range obtained in Ref. [42]. Finally, for this case, we observed that we have found a lower bound for the parameter C_ϕ from the ratio $\Gamma/3H > 1$ and an upper bound from BICEP2 experiment or Planck satellite.

For the case in which $m = 1$ ($\Gamma \propto T$), we find that the tensor-scalar ratio $r \simeq 0$, and the model is disproved from BICEP2, since $r = 0.2^{+0.07}_{-0.05}$, with $r = 0$ disproved at 7.0σ. However, previous CMB observations from the Planck satellite and other CMB experiments yielded only an upper limit for the ratio $r < 0.11$ (at 95% C.L.). In this form, for the case $m = 1$ we numerically obtain that the parameter $C_\phi > 0.05$ is well supported by the strong regime, in which $\Gamma/3H > 1$. Also, we observe that when we increase the value of the parameter C_ϕ, the value of the tensor-to-scalar ratio $r \simeq 0$. In particular, for the value $C_\phi = 0.05$ corresponds to $(\frac{\Gamma}{3H})|_{N=60} \simeq 1.5$, the ratio $(\frac{T}{H})|_{N=60} \simeq 88$ and the tensor-to-scalar ratio $r|_{n_s=0.96} \simeq 0.002$.

Also, in the strong regime we observe that for the cases in which $m = 0$ and $m = -1$, the models are disproved from observations; since spectral index $n_s > 1$, these models do not work.

Analogous to the case of the weak dissipative regime, we also study the evolution of the ratio T/H on the number of e-folds N for the strong dissipative regime. For the special case $m = 3$, we find that the constraint for the parameter C_ϕ, given by $10^8 < C_\phi < 10^{10}$ is well supported by the condition for the warm inflation scenario in which $T/H > 1$ when the number of e-folds $N \geq 60$. Here, we numerically utilize Eqs. (2), (13), (27), and (35) together with the same parameters of the Fig. 4. In particular, for the value $C_\phi = 10^8$ we find that the value of ratio $(\frac{T}{H})|_{N=60} \simeq 49.5$, for the value $C_\phi = 10^9$ corresponds to $(\frac{T}{H})|_{N=60} \simeq 311.1$ and for $C_\phi = 10^{10}$ it corresponds to $(\frac{T}{H})|_{N=60} \simeq 1104.3$.

IV. CONCLUSIONS

In this paper we have studied the intermediate inflationary model in the context of warped DGP-warm inflation. In the slow-roll approximation, we have obtained analytic solutions of the equations of motion, during the weak and strong regime, for a general form of the dissipative coefficient. For the dissipative coefficient $\Gamma(T, \phi) = C_\phi T^m/\phi^{m-1}$, we have investigated the specific values $m = 3$, $m = 1$, $m = 0$, and $m = -1$. In our model, we have found analytical expressions for the corresponding effective potential, power spectrum, scalar spectrum index, and tensor-to-scalar ratio. From these quantities, we have obtained, in both regimes, constraints on the parameters of the model from the BICEP2 experiment and Planck, where we have consider the constraint on the $r - n_s$ plane.

On the other hand, we have obtained a constraint for the value of the parameter C_ϕ from the both regimes, i.e., $\Gamma/3H < 1$ or $\Gamma/3H > 1$, and also we have found a new constraint for the parameter C_ϕ from the condition the warm inflation $T > H$. In the weak dissipative regime, we have obtained an upper bound for the parameter C_ϕ from BICEP2, Planck and a lower bound from the condition for the warm inflation $T > H$, and we have observed that when we decrease the value of m, the value of the parameter C_ϕ also decreases. For the strong dissipative regime, the model only works for the case $m = 3$, i.e., $\Gamma \propto T^3/\phi^2$. Here, we have obtained a lower bound for the parameter C_ϕ from the ratio $\Gamma/3H > 1$, and an upper bound from the BICEP2 experiment and Planck data. For the case in which $m = 1$ ($\Gamma \propto T$), we have found that $r \simeq 0$, and the model is disproved by BICEP2. However, from the Planck satellite in which $r < 0.11$, we have found a lower bound for the parameter C_ϕ. Finally, we have observed that for the cases in which $m = 0$ and $m = -1$, the models are disproved by observations, since the spectral index $n_s > 1$, and these models do not work.

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